MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

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MEETING OF 1947 DECEMBER 12

Professor W. M. H. Greaves, President, in the Chair

The President announced that the following reply had been received from the Secretary of State for Home Affairs to the Loyal Address sent to His Majesty the King on the occasion of the wedding of H.R.H. Princess Elizabeth :—

Home Secretary.

Whitehall,
London, S.W.1.
1st December, 1947.

Sir,

I have had the honour to lay before The King the loyal and dutiful Address of the Royal Astronomical Society on the occasion of the marriage of Her Royal Highness the Princess Elizabeth to His Royal Highness the Duke of Edinburgh.

I have it in command from The King to convey to you, as President of the Society, the sincere thanks of Their Majesties for the kindness and cordiality of these congratulations and good wishes for the future happiness and prosperity of Their Royal Highnesses, which they were specially pleased to receive on this happy occasion.

I am, Sir,
Your obedient Servant,
J. CHUTER EDE.

The President,
Royal Astronomical Society.

The election by the Council of the following Fellows was duly confirmed :—

John Werner Abrams, Ph.D., Wesleyan University, Middletown, Connecticut, U.S.A.;

Catharine E. Barry, Hayden Planetarium, American Museum of Natural History, Central Park West, New York, U.S.A.;

Frank George Griffith Carr, National Maritime Museum, Greenwich, London, S.E.10;

Rev. Herbert Gilgryst, Clifton Vicarage, York;

Herbert Victor Griffiths, B.B.C. Measurement and Receiving Station, Tatsfield;

Karel Hujer, D.Sc., University of Chattanooga, Tennessee, U.S.A.;

Anthony William Jewiss, 146 Lennard Road, Beckenham, Kent;

Donald H. Menzel, Ph.D., Harvard College Observatory, Cambridge, Massachusetts, U.S.A.;

Sydney William Royle Mottram, 30 Hartington Road, Chorlton-cum-Hardy, Manchester;

Robert Naismith, M.I.E.E., 41 Downs Road, Slough, Buckinghamshire;
B. V. Raman, D.Sc., Ph.D., Seshadripuram, P.O. Malleswaram, Bangalore, India;
Rev. John Gammon Read, The Parsonage, Wortwell, Harleston, Norfolk;
Alfred Athelstan Smart, 2 Ferntower Road, Highbury, London, N.5; and
Samuel Tolansky, D.Sc., Ph.D., 16 Sheen Common Drive, Richmond, Surrey.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Gerald Balmforth, Commercial Arcade, Guernsey, Channel Islands (proposed by W. Kilshaw);
Donald Archibald Campbell, Alfarda, Lybury Lane, Redbourn, Hertfordshire (proposed by J. P. M. Prentice);
The Rt. Hon. Lord Cooper, P.C., O.B.E., K.C., 11 Parliament Square, Edinburgh (proposed by W. M. H. Greaves);
Martin Ryle, Cavendish Laboratory, Cambridge (proposed by A. D. Thackeray);
Edward Denton Sherlock, 11 Ballbrooke Avenue, Didsbury, Manchester (proposed by H. L. Dilks); and
Sydney Ernest Williams, Ph.D., The University, Perth, Australia (proposed by R. v. d. R. Woolley).

Sixty-six presents were announced as having been received since the last meeting, including :—

C. M. Botley, *Polar Lights* (presented by the author);
G. F. S. Hills, *Formation of Continents by Convection* (presented by the author); and
H. T. Stetson, *Sunspots in Action* (presented by the author).

At a Special General Meeting of Fellows, held immediately preceding the Ordinary Meeting, additions and amendments to the Bye-laws designed to provide for the institution of Junior Membership of the Society were proposed severally by Professor W. H. McCrea, Secretary, on behalf of the Council, and seconded by Dr A. D. Thackeray, Secretary, in each case :—

SECTION XXI.—*Of Junior Members*

86. Any person under the age of 25 desirous of admission into the Society as a Junior Member must be proposed and recommended agreeably to the Form No. 4 in the Appendix hereto; wherein must be inserted the Christian name, surname, date of birth, place of training, or occupation, and usual place of residence of the candidate; and which form must be subscribed by at least one Fellow or Associate who must certify his personal knowledge of such candidate.

87. Every recommendation of a proposed Junior Member must be delivered to one of the Secretaries, and read at one of the ordinary meetings of the Society.

88. The election of candidates proposed as Junior Members shall rest with the Council, subject to confirmation by the Fellows in meeting assembled. Such election shall be decided at the first ordinary meeting of Council after the reading of the recommendation as prescribed by the preceding Bye-law.

89. At the next ordinary meeting of the Society a list of the candidates so elected shall be read and put to the meeting for confirmation, when, unless any Fellow demands a ballot, the election of all candidates in such list shall be deemed to be confirmed. If a ballot is demanded, it shall be taken at the close of the same meeting when each of the candidates shall be balloted for separately.

90. Whenever a person is elected a Junior Member, one of the Secretaries shall inform him of the same, by letter, as soon as possible; and shall at the same time send him a copy of this Section XXI of the Bye-laws, and a card announcing the days at which ordinary meetings are held during the session.

91. Every person, elected a Junior Member, shall on payment of an annual contribution of one guinea be entitled to attend all ordinary meetings of the Society and to receive the *Monthly Notices* of the Society, or the *Geophysical Supplement*. The Junior Member shall also have the right to receive other publications of the Society at a reduced cost and the right to use the Library. The Junior Member shall not be a corporate member of the Society, and shall therefore have neither the right to confirm the election of Fellows, Associates or Junior Members, nor the right to attend the Annual General Meeting or Special General Meetings of the Society, nor the right to take part in the election of Officers and Council, nor finally shall he himself be eligible to hold any office in the Society.

92. Every person, elected a Junior Member, shall pay his first annual contribution within two calendar months of the day of his election, otherwise his election shall be void. On payment of his first annual contribution the Junior Member shall receive the *Monthly Notices* (or the *Geophysical Supplement*) for the whole of the year in which his election took place, and his subsequent annual contributions will become due and payable in advance on the 1st day of January of the subsequent years. Delivery of the *Monthly Notices* (or the *Geophysical Supplement*) to the Junior Member in these subsequent years shall commence on receipt of his annual contribution, and shall be continued until he has received the whole of the numbers for that year. Whenever a Junior Member shall be one year in arrear in the payment of his annual contribution, he shall cease to enjoy the rights of a Junior Member, and shall be deemed to have resigned.

93. A Junior Member may retain his status and privileges for a period of four years from the 1st day of January of the year of his election, or until the end of the year during which he attains the age of twenty-five, whichever is earlier. At any time before his automatic retirement as a Junior Member and provided his annual contribution has been paid, a Junior Member may be proposed for admission into the Society as a Fellow, in accordance with Section IV of these Bye-laws. If elected, his contribution shall be that determined by Section V of these Bye-laws, except that he shall not be required to pay the usual admission fee of two guineas.

94. Nothing in the present Section XXI of these Bye-laws shall be deemed to hinder any person being proposed for admission as a Fellow, who though eligible has not taken advantage of the possibility of becoming a Junior Member.

Section XXI and Bye-law 86 shall be amended to Section XXII and Bye-law 95 respectively.

Bye-law 16. At the next ordinary meeting (the Annual General Meeting included) a list of the candidates so elected shall be read and put to the meeting for confirmation, when, unless any Fellow demands a ballot, the election of all candidates in such list shall be deemed to be confirmed. If a ballot is demanded, it shall be taken at the close of the same meeting, when each of the candidates shall be balloted for separately.

shall be amended to read as follows :—

Bye-law 16. At the next ordinary meeting of the Society a list of the candidates so elected shall be read and put to the meeting for confirmation, when, unless any Fellow demands a ballot, the election of all candidates in such list shall be deemed to be confirmed. If a ballot is demanded, it shall be taken at the close of the same meeting, when each of the candidates shall be balloted for separately.

The Secretary explained the reasons for the additions and amendments to the Bye-laws, and the motions were carried unanimously.

SPHERICALLY SYMMETRICAL MODELS IN GENERAL RELATIVITY

H. Bondi

(Received 1947 August 5)

Summary

The field equations of general relativity are applied to pressure-free spherically symmetrical systems of particles. The equations of motion are determined without the use of approximations and are compared with the Newtonian equations. The total energy is found to be an important parameter, determining the geometry of 3-space and the ratio of effective gravitating to invariant mass. The Doppler shift is discussed and is found to contain both the velocity shift and the Einstein shift combined in a rather complex expression.

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1. *Introduction.*—The field equations of the general theory of relativity are very complex. The only non-static solutions which have so far been obtained are either approximations or are of the cosmological type. Since approximate solutions apply only in the cases where the field is almost Newtonian, their use in pointing out intrinsic consequences of the theory is somewhat restricted. Similarly cosmological solutions suffer from the disadvantage that the spatial part of space-time is supposed to be homogeneous and isotropic. Therefore it is often difficult, owing to the lack of independent variables, to disentangle the causes of various effects.

The main purpose of the present paper is to derive the equation of motion and to describe various properties of pressure-free spherically symmetrical systems. A rigorous solution of the field equations has been obtained and it is hoped that the model presented can be of use in illustrating and clarifying various points of interest in the theory.

The work may be considered to be an extension of the work of McCrea, McVittie and Lemaitre * on the problem of condensation. The applicability of the work of these authors is somewhat restricted by the fact that they consider only small deviations from an Einstein universe. The work in the present paper is more general in that the system need approximate neither to a Newtonian system nor to an Einstein universe, but is more restricted in its assumptions of zero pressure and of spherical symmetry. Since the general trend in recent work in this field seems to be towards pressure-free systems the former assumption is not as restrictive as might appear at first sight.

Tolman † considers a system identical with ours and derives equivalent equations of motion. His discussion of it however is concerned with properties of the system very different from those to which detailed consideration is given in this paper.

* W. H. McCrea and G. C. McVittie, *M.N.*, **91**, 128, 1930; **92**, 7, 1931; G. C. McVittie, *M.N.*, **91**, 274, 1931; **92**, 500, 1932; **93**, 325, 1933; G. Lemaitre, *M.N.*, **91**, 483, 490, 1931. In the last named paper Lemaitre studies a problem very closely related to ours and many equations given in the appendix can be found in the paper.

† R. C. Tolman, *Proc. Nat. Acad. Sci.*, **20**, 169, 1934.

The assumption of spherical symmetry supplies us with a model which lies between the completely homogeneous models of cosmology and the actual universe with its irregularities. In this sense an advance has been made which, though small, suffices to show up a number of significant features such as the shift of the spectral lines discussed in Section 7 and the connection between the geometry of 3-space and the energy of the spherical shells of matter (Section 5).

The equation of motion obtained is very simple, and is, but for a different interpretation of the constants, identical with the Newtonian equation of energy (Section 4). The extreme simplicity of this result is a very attractive feature of the theory.

In connection with the problem of the equations of motion in general relativity, it is interesting to observe that the postulate of the motion of matter along geodesics does not lead to any contradictions with the field equations in our system, which seems to be more complicated than any other system yet examined without the use of approximations.

The question of boundary conditions at infinity does not arise in our model; the condition that the centre of the system is an ordinary point (we exclude point masses) is found to be sufficient to determine the solutions of the equations uniquely.

Gravitational units are used throughout the paper, i. e. the velocity of light and the constant of gravitation are put equal to unity.

The author wishes to express his gratitude to Mr F. J. Dyson and to Professor W. H. McCrea for many helpful suggestions.

2. *The Metric.*—We now proceed to give a list of the assumptions made in order to specify the system.

(i) The system is and remains spherically symmetrical, i. e. the mass density and particle velocity are functions of a radial coordinate r and a time coordinate t only, and the motion of each particle is purely radial.

(ii) Each particle moves under the influence of gravity only. This implies that there are no electromagnetic forces acting on the particles and that there are no pressures.

(iii) The orbits of particles do not intersect. This means that they do not overtake each other. If an imaginary observer moves outwards from the centre he will therefore always pass the particles in the same order irrespective of his starting-time and speed, provided he is faster than any outward-moving particle. The exclusion of intersecting orbits does not lack physical meaning, since if they did intersect pressures would certainly arise.

(iv) The mass density is everywhere finite.

In addition to these assumptions we will also usually put the cosmological constant $\lambda = 0$. This simplifies the mathematics and leaves most of the essential features of the theory unimpaired. All the important formulae will however also be stated in the form they take when λ does not vanish.

In order to define our system of coordinates, suppose that we have a permanent source of light at the centre O of our spherically symmetrical system and surround this source by a small sphere. By assumption (iv) we can use a Galilean system of coordinates in the neighbourhood of O and we can therefore introduce spherical polars on this sphere.

We now define the coordinates θ, ϕ of any event in the following way : consider the ray of light which went from O to the event. The θ, ϕ coordinates of the event are the θ, ϕ coordinates of the point of intersection of this ray and the small sphere.

In order to define the coordinates t, r, we observe that by assumption (ii) and by the fundamental postulates of general relativity our particles move along geodesics. If there are any points of space unoccupied by particles we will imagine these regions to be filled by very fine dust of negligible mass moving so as to satisfy (i), (ii) and (iii). We will assume that this is possible. On this understanding we have a family of non-intersecting geodesics, such that there is one and only one member of the family passing through each point of space-time. Accordingly we can draw a family of hypersurfaces, orthogonal to this family of geodesics, one such hypersurface passing through each point of space-time. We now define these hypersurfaces to be the hypersurfaces of constant t. As is well known in cosmology, the fact that our orthogonal trajectories are geodesics allows us to choose t so that it measures proper time along each member of our family of geodesics.

With assumption (i) and our choice of θ and ϕ, we already have the θ and ϕ coordinates constant along each geodesic. We now choose our r coordinate so that it too is constant along each geodesic and so as to make the surfaces of constant r orthogonal to the other coordinate surfaces. This is evidently possible. We also specify that r is positive and that roughly speaking it increases with distance from the origin. More precisely a point $(t_1, r_1, \theta_1, \phi_1)$ is assumed to be between the origin O and a point $(t_1, r_2, \theta_1, \phi_1)$ if, and only if, $r_1 < r_2$. Otherwise r is arbitrary.

In concluding this definition of our system of coordinates it must be mentioned that if assumption (iii) (non-intersection) does not hold throughout all space-time, but only throughout a finite or infinite region of space-time including O for some period, then we can still introduce our coordinates in at least part of that region and the theory will be valid there.

It follows from our assumption and our definition of the coordinates that our metric is

$$ds^2 = dt^2 - X^2(r,t)\,dr^2 - Y^2(r,t)(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{1}$$

where $X(r,t)$ and $Y(r,t)$ are functions of r and t only.

Accordingly the metric tensor $g_{\mu\nu}$ is *

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -X^2 & 0 & 0 \\ 0 & 0 & -Y^2 & 0 \\ 0 & 0 & 0 & -Y^2\sin^2\theta \end{bmatrix}. \tag{2}$$

The field equations of general relativity establish a connection between this tensor and the energy tensor $T^{\mu\nu}$. Since our system is without pressure

$$T^{\mu\nu} = \rho \frac{dx^\mu}{ds}\frac{dx^\nu}{ds}, \tag{3}$$

where ρ is the invariant density and dx^μ/ds is the velocity of the matter. In our system of coordinates each particle moves in a way which keeps its r, θ, ϕ coordinates constant and makes the time component of its velocity equal to unity. Accordingly

$$T^{\mu\nu} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{4}$$

* We use the suffixes (0, 1, 2, 3) for (t, r, θ, ϕ) in that order.

ρ is of course a function of r and t.

We will find it advantageous to use in our work not ρ but M, the sum of the invariant masses of all the particles with radial coordinate less than r. By our non-intersection assumption this mass depends on r only, so that $M = M(r)$.

Clearly

$$M(r) = \int_0^r dr \int_0^\pi d\theta \int_0^{2\pi} d\phi \, T\sqrt{-g} = 4\pi \int_0^r dr \rho XY^2, \tag{5}$$

so that

$$\frac{dM}{dr} = 4\pi\rho XY^2. \tag{6}$$

Then the field equations of general relativity take the form *

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = -8\pi T_{\mu\nu} = \begin{cases} -\dfrac{2M'(r)}{XY^2} & (\mu = \nu = 0), \\ 0 & \text{(otherwise)}, \end{cases} \tag{7}$$

where $R_{\mu\nu}$ is the Einstein tensor.

3. *The definition of Distance.*—Before we discuss the solution of the field equations, it will be desirable to investigate the main features of the propagation of light in our system, so that we can define distance.

Astronomically, the most important definition of distance is probably luminosity distance.† The apparent luminosity of a source of known absolute luminosity is measured and is corrected for Doppler shift. The distance of the source is then defined as being proportional to the square root of the ratio of absolute and corrected apparent luminosity. As a result of the researches of Tolman, v. Laue and Robertson ‡, it is known that in a homogeneous universe the square of the Doppler shift has to be taken as the correcting factor. We shall now show that in our spherically symmetrical universe with the standard source at its centre, it is still correct to use the square of the Doppler shift, and that the luminosity distance of this standard source for an observer at (t, r, θ, ϕ) is $Y(r, t)$.

By the definition of our system of coordinates a ray of light travelling outwards from the centre satisfies the equations

$$\frac{dt}{dr} = X(r, t), \tag{8}$$

$$\theta = \text{const.}, \qquad \phi = \text{const.} \tag{9}$$

Consider now two rays with the same θ and ϕ values, and let the equation of the first ray be

$$t = T(r), \tag{10}$$

while the equation of the second ray is

$$t = T(r) + \tau(r). \tag{11}$$

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* In the appendix a more general problem is also discussed, viz. what system of pressures is compatible with the motion of particles along our geodesics. While such a system of pressures is in general more or less arbitrary, it is interesting to note that the assumption of isotropic pressure (i. e. $T_1^1 = T_2^2 = T_3^3$) is easily seen to imply that the pressure is a function of our time coordinate t only.

† W. H. McCrea, *Z. f. Ap.*, **9**, 290, 1935.

‡ R. C. Tolman, *Proc. Nat. Acad. Sci.*, **16**, 511, (para. 6), 1930; M. v. Laue, *Z. f. Ap.*, **12**, 208, 1936; H. P. Robertson, *Z. f. Ap.*, **15**, 69, 1937.

We shall also assume that $\tau(r)$ is small. Then by (8)

$$\frac{dT(r)}{dr} = X\{r, T(r)\}, \quad \frac{d\tau(r)}{dr} = \tau(r)\left(\frac{\partial X}{\partial t}\right)_{r, T(r)}. \tag{12}$$

This result gives the equation of a ray and the rate of variation of $\tau(r)$ along the ray. If we take $\tau(0)$ to be the period of oscillation of some spectral line at the origin, the Doppler shift in the Hubble-Tolman notation will be

$$\frac{\tau(r)}{\tau(0)} = \frac{\nu_0}{\nu} = 1 + z. \tag{13}$$

Considering now z not as a function of r along a ray but as a function of t and r throughout space-time, we see that it satisfies the partial differential equation

$$\frac{\partial z}{\partial r} + X\frac{\partial z}{\partial t} = (1+z)\frac{\partial X}{\partial t}; \quad z = 0 \text{ at } r = 0. \tag{14}$$

A more detailed analysis of equations (12) will be given in Section 7; for our present purposes equation (14) is sufficient.

In order to investigate the variation of intensity with distance, we adopt Robertson's procedure.*

Consider an observer at $t_1, r_1, \theta_1, \phi_1$. For measurements in his neighbourhood he will use a local Galilean system $\bar{t}, \bar{x}, \bar{y}, \bar{z}$ with

$$\left.\begin{aligned} &\bar{t} = t - t_1, \qquad \bar{x} = X(r_1, t_1)(r - r_1), \qquad \bar{y} = Y(r_1, t_1)(\theta - \theta_1), \\ &\bar{z} = Y(r_1, t_1)(\phi - \phi_1)\sin\theta_1, \\ &ds^2 = d\bar{t}^2 - d\bar{x}^2 - d\bar{y}^2 - d\bar{z}^2. \end{aligned}\right\} \tag{15}$$

Assuming the wave-length of light to be minute compared with the dimensions of our system, the wave coming from the origin will appear to him to be plane, and hence his measurement of the electromagnetic energy tensor will give

$$\overline{E^{00}} = \overline{E^{01}} = \overline{E^{11}} = U \quad \text{(say)}, \tag{16}$$

while all other components vanish. U will be his measurement of the apparent luminosity of the source at O.

In our usual system of coordinates

$$E^{00} = U, \quad E^{01} = \frac{1}{X}U, \quad E^{11} = \frac{1}{X^2}U, \tag{17}$$

while all other $E^{\mu\nu}$ vanish. Applying the relativistic conservation law $(E^{\mu\nu})_\nu = 0$, we obtain one equation for U, viz.†,

$$\frac{\partial U}{\partial r} + X\frac{\partial U}{\partial t} + 2\left(\frac{\partial X}{\partial t} + \frac{X}{Y}\frac{\partial Y}{\partial t} + \frac{1}{Y}\frac{\partial Y}{\partial r}\right) = 0. \tag{18}$$

Putting

$$U = \frac{C}{Y^2(1+z)^2}, \tag{19}$$

and making use of (14), we find

$$\frac{\partial C}{\partial r} + X\frac{\partial C}{\partial t} = 0. \tag{20}$$

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* H. P. Robertson, *Z. f. Ap.*, **15**, 69, 1937.

† The three index symbols are listed in the appendix (equation (3)).

This means that C is constant along each ray. If the source at the centre does not vary, then C will be constant throughout all space-time. Hence, by (19), Y can be found by observing the apparent luminosity of a source of known absolute luminosity, and applying the square of the Doppler shift as correcting factor.

The significance of this result is twofold : First we note that the important concept of luminosity distance is equivalent to our Y, which, as we shall see, is mathematically the most convenient dynamical variable in our system.

Secondly it is interesting to find that in our extension of the previously known use of the correcting factor, we can still use the whole of the Doppler shift, the effects of the Einstein and velocity shifts being indistinguishable.

Two other definitions of distance may be mentioned. If an observer at O measures the distance of an object of known size at (t, r, θ, ϕ) by measuring its apparent size, he will evidently obtain Y. If an observer at (t, r, θ, ϕ) measures his distance from O by measuring the parallax of O, it can be shown that this result will be $Y(X/Y')^{\frac{1}{2}}$. As we shall see in the next section, X/Y', the square of the ratio of this "distance" to Y, is, as a consequence of the field equations, a function of r only, i. e. it is a constant for every observer moving with the particles.

Finally, consider a test particle following an arbitrary geodesic. It can easily be shown that we can turn the system of coordinates so that the geodesic lies entirely in the surface $\theta = \pi/2$, and then there is an integral of angular momentum which is

$$Y^2 \frac{d\phi}{ds} = \text{const.}$$

Again Y takes the part of the classical radius.

We see therefore that Y is a variable of considerable significance occupying in many ways a position corresponding to the classical concept of distance. Accordingly, we shall refer to $Y(r, t)$ as the distance of the particle (r, θ, ϕ) from the origin at time t and, since t measures the particle's proper time, we shall refer to $\partial Y/\partial t$ as its velocity.

4. *The Equations of Motion.*—The equations (7) are discussed in the appendix, where it is shown that they are equivalent to

$$X = \frac{1}{W(r)} \frac{\partial Y}{\partial r}, \tag{21}$$

$$\left(\frac{\partial Y}{\partial t}\right)^2 = W^2(r) - 1 + \frac{2}{Y} \int_0^r M'(r) W(r)\, dr, \tag{22}$$

where $W(r)$ is an arbitrary function of r. (21) merely expresses X in terms of Y, while (22) supplies us with the equation of motion.*

In order to compare (22) with the Newtonian approximation, we have to consider the case of small velocities and small masses. Accordingly W will be near unity and we put

$$W^2(r) = 1 + 2E(r), \tag{23}$$

where E is small. Then, neglecting the product of M' and E, we have

$$\frac{1}{2}\left(\frac{\partial Y}{\partial t}\right)^2 - \frac{M(r)}{Y} = E(r), \tag{24}$$

* Equation (22) can be integrated again. The integration is carried out in the appendix, but the result is of little importance for us.

which is identical with the Newtonian equation of energy, E representing the total energy per unit mass. The exact equation (22) may be re-written

$$\frac{1}{2}\left(\frac{\partial Y}{\partial t}\right)^2 - \frac{1}{Y}\int_0^r dr\, M'(r)\{1+2E(r)\}^{\frac{1}{2}} = E(r), \tag{25}$$

and we see, then, that with our definition of distance and velocity the only difference between our equation and the Newtonian equation is that the effective gravitating mass is not the invariant mass. The most interesting point of this result is that the ratio of effective gravitating mass and invariant mass depends not on the kinetic energy but on the total * energy $E(r)$. This suggests that the total energy and hence the potential energy have a rather greater significance in general relativity than hitherto supposed, a point which will be more fully discussed in the next section.

The acceleration $\partial^2 Y/\partial t^2$ can be determined by differentiating (25). The result is

$$\frac{\partial^2 Y}{\partial t^2} = -\frac{1}{Y^2}\int_0^r M'(r)\, dr\{1+2E(r)\}^{\frac{1}{2}}. \tag{26}$$

Hence we still have an inverse square law. However, it is now impossible to derive (25) from (26) without ambiguity, since $E(r)$ the constant of integration is already contained in (26).

An important similarity between our equations and those of the Newtonian theory is that in both theories the spherical shells of matter further away from O than a particle P do not effect the motion of P at all.

We are now in a position to discuss how $W(r)$ (and hence $E(r)$) are determined. As is usual in dynamics, our system will be fully defined only if, at some instant, the positions and velocities of all particles are given. It will be seen that in our case it is unnecessary that the velocities and positions of different particles are given at instants related in any particular way.

Let $t=t_0(r)$ prescribe the value of t at which the position and velocity of the particles at r are given as well as the density in their neighbourhood. We must of course assume that $t_0(r)$ is a single valued function. Let

$$Y\{r, t_0(r)\} = R(r), \tag{27}$$

$$\left(\frac{\partial Y}{\partial t}\right)_{r,\, t_0(r)} = V(r), \tag{28}$$

and let us assume that the mass distribution is given by giving $M(r)$. (27) is really only an equation defining r.

Then consider the equation of motion (22) at each point at the moment $t=t_0(r)$. We have

$$V^2 = W^2 - 1 + \frac{2}{R}\int_0^r W(r)M'(r)\, dr. \tag{29}$$

Multiplying by R and differentiating (as we are allowed to do, since each function in (29) is a function of r only),

$$RW\frac{dW}{dr} + \tfrac{1}{2}W^2\frac{dR}{dr} + WM' = \tfrac{1}{2}\frac{d}{dr}\{R(V^2+1)\}. \tag{30}$$

This first-order equation combined with the boundary condition $W=1$ at $r=0$, determines W. ($W=1$ at $r=0$ because, by our assumptions $V(0)=0$ and

* E. T. Whittaker, *Proc. Roy. Soc.*, A., **149**, 384, 1935, obtains similar results.

$M'(r) = o(r^2)$ near $r = 0$). It is easily proved that the equation and the boundary condition determine W uniquely in spite of the singularity at $r = 0$.

If we are not given $M(r)$ but $\rho(r, t_0)$, then (29) takes the form

$$V^2 = W^2 - 1 + \frac{8\pi}{R}\int_0^r \rho\{r, t_0(r)\}\, dr\, R^2 \left\{\frac{dR}{dr} - V\frac{dt_0}{dr}\right\}, \tag{31}$$

and this immediately determines W. The simplicity of this equation is a direct consequence of (6) and (21).

Some remarks must be made about the sign of W. At $r = 0$ $W = 1$. If (as we will assume) V is continuous, then W must be continuous and hence cannot change sign without vanishing at some r. By the definition of our metric $W = 0$ constitutes an impenetrable barrier, since $ds^2 = -\infty$ for any dr. It might be argued that this could be avoided if $Y' = 0$ at the same point. It can however be easily proved from the equation of motion that if $W(r)$ has a nth order zero at some r, then Y' may have a permanent $(n-1)$th order zero there but not a permanent nth order zero.

Hence in all the parts of our system which are connected with the origin, we must have $W > 0$.

The theory can easily be extended so as to include the cosmological constant λ. As is well known, the field equations with λ are

$$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R + \lambda g_{\mu\nu} = -8\pi T_{\mu\nu}. \tag{32}$$

With these field equations we still have

$$X = \frac{1}{W}\frac{\partial Y}{\partial r}, \tag{33}$$

but the equation of motion is now

$$\left(\frac{\partial Y}{\partial t}\right)^2 = \tfrac{1}{3}\lambda Y^2 + W^2 - 1 + \frac{2}{Y}\int_0^r WM'\, dr. \tag{34}$$

This equation brings out very clearly the non-classical character of the λ expansion of large regions. No term corresponding to the λ term can be found in the Newtonian approximation.

5. *Potential Energy.*—One of the most remarkable feats of the general theory of relativity is that the laws of conservation of mass, energy and momentum are combined in the law of conservation of the tensor $T^{\mu\nu}$ and that hence it is the purely kinetic energy which is conserved. The potential energy of Newtonian theory is relegated to the position of a pseudo-tensor which can be made to vanish by a suitable choice of the system of coordinates. It is also well known that in many systems which are approximately Newtonian, potential energy is directly connected with g_{00} if the most obvious system of coordinates is used.

As will be shown in this section, total (and hence potential) energy occupies a significant position in our theory too, although our system need not be approximately Newtonian.

In Section 2 space-time was divided into space and time in a very significant way. This division (by choosing the surfaces $t = \text{const.}$ to be orthogonal to the world lines of the particles) is possible whenever the particles follow non-intersecting geodesics and is independent of the assumption of spherical symmetry. This division is of physical significance, since it is determined by the orbits of the particles. Accordingly some significance can be attached to the 3-space so defined.

In our model this 3-space has the metric

$$d\sigma^2 = X^2\,dr^2 + Y^2(d\theta^2 + \sin^2\theta\,d\phi^2), \tag{35}$$

where

$$X = \frac{1}{W(r)}\frac{\partial Y}{\partial r}.$$

Since we are dealing with a fixed time section, Y may be regarded as a function of r only. Moreover it is a monotonic function of r and hence may be introduced as coordinate.

Then
$$d\sigma^2 = (dY)^2/H^2 + Y^2(d\theta^2 + \sin^2\theta\,d\phi^2), \tag{36}$$

where
$$H(Y) = W(r).$$

The Riemann-Christoffel tensor is easily seen to be given by

$$\left.\begin{aligned} &R_{121}{}^2 = R_{131}{}^3 = -\frac{1}{HY}\frac{dH}{dY}, \quad R_{232}{}^3 = 1 - H^2, \\ &R_{122}{}^3 = R_{121}{}^3 = R_{132}{}^3 = 0. \end{aligned}\right\} \tag{37}$$

Hence the Einstein tensor is

$$\left.\begin{aligned} &R_1^1 = 2\frac{H}{Y}\frac{dH}{dY}, \quad R_2^2 = R_3^3 = \frac{H^2-1}{Y^2} + \frac{H}{Y}\frac{dH}{dY}, \\ &R = \frac{2}{Y^2}\frac{d}{dY}\{Y(H^2-1)\}. \end{aligned}\right\} \tag{38}$$

The most striking consequence of these equations is that the 3-space is flat if, and only if, $H=1$, i.e. $W=1$. If $W>1$ and does not vary too rapidly (or more exactly if $Y(H^2-1)$ is an increasing function of Y), then the curvature of the 3-space is positive, while in the opposite case it is negative.

It should be clearly understood that these statements need not be applied to the whole of space but may be applied to any group of particles occupying a finite interval of r. If e. g. all the particles in the range $r_1 \leqslant r \leqslant r_2$ have zero total energy, then they are embedded in a flat section of 3-space extending (at least) from r_1 to r_2.

Hence the curvature of the 3-space is entirely determined by the total energy of the particles. In our model, total (and hence potential) energy has a direct geometrical significance. We must remember that our division of space and time, while by no means invariant, is a physically significant division.

In addition to this geometrical interpretation of W (i. e. of total energy), W is also (as mentioned in Section 3) the ratio of effective gravitating to invariant mass.

We see, therefore, that we can, in our model, assign a definite place to total (and hence potential) energy even in cases far removed from nearly Newtonian ones.

6. *Cosmological Models.*—Our theory can easily be linked with certain cosmological models.* In these models the 3-space is supposed to be homogeneous and isotropic and this is clearly a special case of spherical symmetry. On the other hand, our theory deals only with pressure-free systems, so that we see that the pressure-free models of relativistic cosmology must be special cases of our models and we proceed to derive them.

Evidently ρ is a function of the time only and hence by (6) and (21)

$$\int_0^r WM'\,dr = \frac{4\pi}{3}\rho Y^3. \tag{39}$$

* *Cf.* H. P. Robertson's article "Relativistic Cosmology", *Rev. Mod. Phys.*, **5**, No. 1, 1933.

It also follows from our expression for the curvature of the 3-space (which must be independent of position) that

$$W^2(r) = 1 + Y^2 f(t).$$

Therefore Y must be a product of a function of t only and a function of r only, so that we may put

$$Y = rg(t) \tag{40}$$

and

$$W^2 = 1 - kr^2, \tag{41}$$

and hence

$$\dot{g}^2 = -k + \tfrac{1}{3}g^2\{\lambda + 8\pi\rho(t)\}. \tag{42}$$

Since $\int_0^r WM'\,dr$ is a function of r only, it follows from (39) and (40) that ρg^3 is a constant. Therefore (42) takes the form

$$3g(\dot{g}^2 + k) - \lambda g^3 = 8\pi g^3\rho = \text{constant}.$$

The various cases of relativistic cosmology arise for different values of the constants. The metric is given by

$$ds^2 = dt^2 - [g(t)]^2\left\{\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2\sin^2\theta\, d\phi^2\right\}. \tag{43}$$

The substitution $r = R/(1 + \frac{1}{4}kR^2)$ turns (43) into the more familiar form

$$ds^2 = dt^2 - \left\{\frac{g(t)}{1 + \frac{1}{4}kR^2}\right\}^2\{dR^2 + R^2\,d\theta^2 + R^2\sin^2\theta\,d\phi^2\}.$$

7. *The Doppler Shift.*—In Section 3 we found that $Y(r, t)$ had direct physical significance as the value an observer at (r, t) would assign to his distance from O as a result of his measurement of apparent luminosity of a source at O corrected for Doppler shift. $\partial Y/\partial t$ was then called his velocity, being the rate of change of distance from O with proper time. Now evidently his measurement of the Doppler shift supplies him with another definition of velocity and the relation between the Doppler shift and his velocity $\partial Y/\partial t$ will be discussed in this section.

We saw in Section 3 that the equation of a ray of light travelling outwards from O was

$$\theta = \text{const.}, \qquad \phi = \text{const.}, \qquad t = T(r),$$

where

$$\frac{dT(r)}{dr} = X\{r, T(r)\}, \tag{44}$$

and that the Doppler shift $1 + z$ satisfied

$$\frac{dz}{dr} = (1 + z)\left(\frac{\partial X}{\partial t}\right)_{r, T(r)} \qquad z = 0 \text{ at } r = 0.$$

Hence the Doppler shift $1 + z_1$ at $r = r_1$, $t = t_1$ is given by

$$\log(1 + z_1) = \int_0^{r_1} dr\left(\frac{\partial X}{\partial t}\right)_{r, T(r)} = \int_0^{r_1}\frac{dr}{W(r)}\left(\frac{\partial^2 Y}{\partial r\,\partial t}\right)_{r, T(r)}, \tag{45}$$

where $T(r)$ is the solution of (44) passing through $r = r_1$, $t = t_1$.

We can find a more significant expression for (45) if $W + Y \neq 0$ (as will usually be the case).* For then $Y\{r, T(r)\}$ is an increasing function of r (its differential

* The meaning of this restriction is discussed at the end of this section.

quotient is $Y' + T'\dot{Y} = Y'(W + \dot{Y})/W$) and hence we are allowed to re-label our r coordinate in such a way that $Y\{r, T(r)\} = r$.

Now let

$$\left(\frac{\partial Y}{\partial t}\right)_{r,\,T(r)} = v(r), \quad \left(\frac{\partial^2 Y}{\partial t^2}\right)_{r,\,T(r)} = f(r). \tag{46}$$

Then expanding Y in a Taylor series in $t - T$ we have

$$Y(r,t) = r + v(r)\{t - T(r)\} + \tfrac{1}{2}f(r)\{t - T(r)\}^2 + \ldots. \tag{47}$$

Equation (44) for T becomes

$$WT' = 1 - vT' \quad \text{or} \quad T' = \frac{1}{W + v}, \tag{48}$$

while, from the equation of motion,

$$v^2 = W^2 - 1 + \frac{2}{r}\int_0^r WM'\,dr, \quad f = -\frac{1}{r^2}\int_0^r WM'\,dr.$$

Substituting into (45) we have

$$\log(1 + z_1) = \int_0^{r_1} dr\,\frac{v' - fT'}{W} = \int_0^{r_1} dr\,\frac{Wv' + vv' - f}{W(W + v)}. \tag{49}$$

Substituting for $vv' - f$ from the equation of motion we have

$$\log(1 + z_1) = \int_0^{r_1} dr \frac{v' + W' + \dfrac{M'}{r}}{v + W} = \log(v_1 + W_1) + \int_0^{r_1} dr \frac{M'}{r(W + v)}, \tag{50}$$

since $v_0 = 0$, $W_0 = 1$.

A little care is required in interpreting this expression. The shift of spectral lines is due to two causes, viz. the velocity shift due to the relative motion of source and observer, and the Einstein shift due to the difference between the potential energy per unit mass at the source and at the observer. The velocity shift is, in our units, just equal to v_1 and is easily identified in (50).

The Einstein shift is of a more complicated type. We have so far identified

$$-\int_0^{r_1} M'W\,dr/r_1,$$

with the potential energy per unit mass at r_1 but this is only true in a very restricted sense. For in bringing a particle from infinity to r_1 we have tacitly assumed (by virtue of the non-intersection hypothesis) that all the spherical shells of matter outside r_1 were moved to their positions from infinity in such a way that they were always beyond our particle.

We obtained the correct equation of motion for our particles with this definition of potential energy only because the particle orbits do not intersect. For a ray of light the situation is radically different, since it passes matter on its way. Accordingly we now require a new definition of potential energy, which we will first obtain in the Newtonian analogue.

There the force per unit mass is $-M/r^2$ and accordingly the difference in potential energy between the origin and r_1 is

$$\int_0^{r_1} \frac{M(r)}{r^2}\,dr = -\frac{M(r_1)}{r_1} + \int_0^{r_1} \frac{M'\,dr}{r}, \tag{51}$$

by integration by parts.

But for small masses and velocities (50) becomes

$$v_1 + W_1 + \int_0^{r_1} \frac{M'}{r}\,dr - 1 = v_1 + \tfrac{1}{2}v_1^2 - \frac{M(r_1)}{r_1} + \int_0^{r_1} \frac{M'\,dr}{r}, \tag{52}$$

since $W-1$ is the energy per unit mass.

The first term on the right-hand side is the ordinary velocity shift, the second term is small compared with it, while the last two terms describe the Einstein shift in terms of the Newtonian potential (51).

We see then that the exact expression (50) gives us an expression for the combination of velocity shift and Einstein shift in our model.

An interesting, but as we have seen not a radically new, point in (50) is the fact that the spectral shift does not only depend on conditions at the source and at the observer but also on the distribution of matter in the intervening space. Note that any empty part of space does not contribute to the integral in (50).

The sign of the velocity shift depends on the sign of v_1, but the Einstein shift is easily seen to be towards the red, at least for reasonably small masses and velocities.

For light proceeding in the opposite direction the velocity shift has the same sign as before, but the Einstein shift changes sign. An analysis similar to the one above gives for light travelling from r_1 to the origin

$$\log \frac{\tau_0}{\tau_1} = -\log(W_1 - v_1) - \int_0^{r_1} \frac{M'\,dr}{r(W-v)}.$$

Finally it might be mentioned that if we re-introduce the cosmological constant λ, no change is made in (50) or any of the subsequent arguments. Hence there is no shift of the spectral lines explicitly due to λ.

In this discussion of Doppler shift we have so far restricted ourselves to the case $W+\dot{Y}>0$. What is the significance of this condition?

As we move along the ray, Y changes and

$$\frac{dY}{dt} = \frac{\partial Y}{\partial t} + \frac{\partial Y}{\partial r}\frac{dr}{dt} = W + \frac{\partial Y}{\partial t}. \tag{53}$$

Hence Y increases along the ray only as long as $W+\dot{Y}>0$. This is due to the curious way light is dragged along by matter (which is also exemplified by equation (48)). If matter is falling into the origin at very high speed it may stop the ray and even reverse its direction. Of course this applies only to the Y picture; the ray continues to reach matter with higher and higher r values but only when this new matter has moved sufficiently near to the origin. In the Y picture there is a barrier to outward-moving rays of light and therefore our definition of Y does not apply beyond this region. This barrier is not nearly as impenetrable as the $W=0$ barrier mentioned in Section 3, since e.g. inward-moving rays of light can easily cross it.

It might be mentioned that this reversal of the direction of light can only occur if we have very high densities throughout very large regions. For if $-\dot{Y} < W < 0$ then by (22)

$$W^2 - 1 + \frac{2}{Y}\int_0^r M'W\,dr > W^2,$$

and hence we must have

$$\int_0^r M'(r)W(r)\,dr = 4\pi\int_0^r \rho Y^2 \frac{\partial Y}{\partial r}\,dr > \tfrac{1}{2}Y. \tag{54}$$

It is a remarkable fact that while inward-moving matter can in such extreme circumstances reverse the direction of an outward-travelling ray, such a ray will always catch up with outward-moving matter (no matter how large its $\dot{Y}$ may be) provided it has not been held up previously by the above-mentioned barrier.

8. *Limitations of the Theory.*—With the exception of the non-intersection hypothesis our assumptions, if correct initially, will necessarily remain correct as the motion progresses. However, a serious limitation of the validity of the theory arises because the system may develop in such a way that the non-intersection hypothesis, although initially true, is later violated.

We have to consider separately the case where the violation of the hypothesis occurs at the origin and the case in which it occurs elsewhere.

If the orbits of particles intersect at any point other than O, this fact would show itself in our notation by Y' vanishing at some point. For by our definition of our metric this would imply that the distance between the particles of two different shells has become zero. Accordingly our equations apply only up to the minimum value of t for which $Y'(r,t)$, considered now as a function of r, has a zero. If however we assume that this intersection does not upset the spherical symmetry, it seems that our theory will remain valid even for larger t for all $r<r_1(t)$, where $r_1(t)$ is the least r at which $Y'(t,r)=0$. Our theory remains valid because the gravitational field at a point is independent of the spherical shells of matter beyond it.

A more serious violation of the non-intersection assumption may occur at O. For if matter near O moves into O it will either pass through O and re-emerge on the other side, its orbits intersecting the orbits of the incoming matter, or a point mass will be formed at O. In both cases the theory breaks down completely, in the first case because of the intersections in the neighbourhood of O, in the second case because of the singularity at O. Although this singularity might formally be included in the equations the breakdown of the physical interpretation of Y would rob the theory of much of its significance.

No such difficulty at O will arise if originally there is a small empty region round O and if the matter nearest to O does not move inwards at first (for then it will never move inwards).

Appendix

Our first step must be to find the energy tensor corresponding to the metric.

$$ds^2=dt^2-X^2(r,t)\,dr^2-Y^2(r,t)\,d\theta^2-Y^2(r,t)\sin^2\theta\,d\phi^2. \tag{1}$$

In this work we will use the field equations including λ.

We put $(t,r,\theta,\phi)=(x_0,x_1,x_2,x_3)$. A dot will denote differentiation with respect to r. Then the Christoffel symbols

$$\Gamma^{\sigma}_{\mu\nu}=\tfrac{1}{2}g^{\sigma\alpha}\left(\frac{\partial g_{\nu\alpha}}{\partial x_\mu}+\frac{\partial g_{\mu\alpha}}{\partial x_\nu}-\frac{\partial g_{\mu\nu}}{\partial x_\alpha}\right) \tag{2}$$

are

$$\left.\begin{aligned}
&\Gamma^1_{11}=\frac{X'}{X},\quad \Gamma^0_{11}=X\dot{X},\quad \Gamma^1_{01}=\frac{\dot{X}}{X},\quad \Gamma^2_{02}=\Gamma^3_{03}=\frac{\dot{Y}}{Y},\quad \Gamma^0_{22}=Y\dot{Y},\\
&\Gamma^2_{12}=\Gamma^3_{13}=\frac{Y'}{Y},\quad \Gamma^1_{22}=-\frac{YY'}{X^2},\quad \Gamma^0_{33}=Y\dot{Y}\sin^2\theta,\\
&\Gamma^1_{33}=-\frac{YY'}{X^2}\sin^2\theta,\quad \Gamma^2_{33}=-\sin\theta\cos\theta,\quad \Gamma^3_{23}=\cot\theta.
\end{aligned}\right\} \tag{3}$$

All other $\Gamma^{\sigma}_{\mu\nu}$ vanish.

The mixed Einstein tensor has components

$$\left.\begin{aligned} R_0^0 &= \frac{\ddot{X}}{X} + 2\frac{\dot{Y}}{Y}, \quad R_1^1 = \frac{\ddot{X}}{X} + 2\frac{\dot{X}\dot{Y}}{XY} + 2\frac{X'Y'}{X^3Y} - 2\frac{Y''}{X^2Y}, \\ R_2^2 &= R_3^3 = \frac{\ddot{Y}}{Y} + \frac{1+\dot{Y}^2}{Y^2} + \frac{\ddot{X}\dot{Y}}{XY} - \frac{1}{X^2}\left(\frac{Y''}{Y} + \frac{Y'^2}{Y^2} - \frac{X'Y'}{XY}\right), \\ R_1^0 &= -X^2R_0^1 = 2\left(\frac{\dot{Y}'}{Y} - \frac{\dot{X}Y'}{XY}\right), \\ R &= 2\frac{\ddot{X}}{X} + 4\frac{\ddot{Y}}{Y} + 4\frac{\dot{X}\dot{Y}}{XY} + 2\frac{1+\dot{Y}^2}{Y^2} - \frac{2}{X^2}\left(2\frac{Y''}{Y} + \frac{Y'^2}{Y^2} - 2\frac{X'Y'}{XY}\right). \end{aligned}\right\} \tag{4}$$

All other components vanish.

Hence the components of the energy tensor are

$$\left.\begin{aligned} \lambda + 8\pi T_0^0 &= 2\frac{\dot{X}\dot{Y}}{XY} + \frac{1+\dot{Y}^2}{Y^2} - \frac{1}{X^2}\left(2\frac{Y''}{Y} + \frac{Y'^2}{Y^2} - 2\frac{X'Y'}{XY}\right), \\ \lambda + 8\pi T_1^1 &= 2\frac{\ddot{Y}}{Y} + \frac{1+\dot{Y}^2}{Y^2} - \frac{Y'^2}{X^2Y^2}, \\ \lambda + 8\pi T_2^2 &= \lambda + 8\pi T_3^3 = \frac{\ddot{X}}{X} + \frac{\ddot{Y}}{Y} + \frac{\dot{X}\dot{Y}}{XY} - \frac{1}{X^2}\left(\frac{Y''}{Y} - \frac{X'Y'}{XY}\right), \\ 8\pi T_1^0 &= -8\pi X^2 T_0^1 = -2\left(\frac{\dot{Y}'}{Y} - \frac{\dot{X}Y'}{XY}\right). \end{aligned}\right\} \tag{5}$$

Owing to the assumption that matter moves without altering its (r, θ, ϕ) coordinates we must have, even in the presence of pressures, $T_1^0 = 0$.

Hence

$$X(r,t) = \frac{1}{W(r)}\frac{\partial Y(r,t)}{\partial r}, \tag{6}$$

where $W(r)$ is an arbitrary function of r.

Then

$$\left.\begin{aligned} \lambda + 8\pi T_0^0 &= \frac{U'}{Y^2Y'}, \\ \lambda + 8\pi T_1^1 &= \frac{\dot{U}}{Y^2\dot{Y}}, \\ \lambda + 8\pi T_2^2 = \lambda + 8\pi T_3^3 &= \frac{1}{2\dot{Y}Y'}\frac{\partial}{\partial r}\left(\frac{\dot{U}}{\dot{Y}}\right), \\ 4\lambda + 8\pi T &= \frac{1}{Y^2Y'}\frac{\partial}{\partial r}\left(Y\frac{\dot{U}}{\dot{Y}} + U\right), \\ \text{where } U &= Y(1+\dot{Y}^2 - W^2). \end{aligned}\right\} \tag{7}$$

As explained in Section 2 this set of equations defines the most general set of pressures which will move the particles along our specified geodesics.

If we assume the pressure to be isotropic then

$$T_1^1 = T_2^2 = T_3^3, \tag{8}$$

and therefore, by (7),

$$\frac{\dot{U}}{Y^2\dot{Y}} = \frac{1}{2YY'}\frac{\partial}{\partial r}\left(\frac{\dot{U}}{\dot{Y}}\right).$$

Hence

$$\frac{\partial}{\partial r}\left(\frac{\dot{U}}{Y^2\dot{Y}}\right) = 0, \tag{9}$$

and so

$$T_1^1 = T_2^2 = T_3^3 = F(t). \tag{10}$$

This means that the pressure is uniform.

We are mainly interested in the case of zero pressure, i.e. $T_1^1 = T_2^2 = T_3^3 = 0$. Then

$$\dot{U} = \lambda Y^2\dot{Y},$$

so that

$$U = \tfrac{1}{3}\lambda Y^3 + S(r). \tag{11}$$

Hence

$$8\pi T_0^0 = 8\pi T = S'(r)/Y^2Y' = S'(r)/XY^2W(r). \tag{12}$$

By (6) of Section 2 this implies

$$S(r) = 2\int_0^r M'(r)W(r)\,dr, \tag{13}$$

since we have $U = 0$ at $r = 0$.

Hence

$$\dot{Y}^2 = W^2 - 1 + \tfrac{1}{3}\lambda Y^2 + \frac{2}{Y}\int_0^r M'(r)W(r)\,dr. \tag{14}$$

In the case $\lambda = 0$ this reduces to

$$\dot{Y}^2 = W^2 - 1 + \frac{2}{Y}\int_0^r M'(r)W(r)\,dr. \tag{15}$$

These equations may be integrated again, viz.

$$t = \int^Y \left\{\tfrac{1}{3}\lambda u^2 + (W^2 - 1) + \frac{2G}{u}\right\}^{-\frac{1}{2}} du, \tag{16}$$

where $$G(r) = \int_0^r M'W\,dr.$$

The lower limit of integration is an arbitrary function of r. It is frequently convenient to have $Y\{r, t_0(r)\} = r$, where $t_0(r)$ is an arbitrary function of r.

Then

$$t = t_0(r) + \int^Y \left\{\tfrac{1}{3}\lambda u^2 + (W^2 - 1) + \frac{2G}{u}\right\}^{-\frac{1}{2}} du. \tag{17}$$

The integral is elliptic unless $\lambda = 0$ when it is elementary. On carrying out the integration for $\lambda = 0$ we find that, when $W > 1$,

$$t = t_0(r) + \frac{2G(r)}{(W^2-1)^{\frac{3}{2}}}\left\{F\left(\frac{2G}{Y(W^2-1)}\right) - F\left(\frac{2G}{r(W^2-1)}\right)\right\}, \tag{18}$$

where

$$F(x) = \frac{\sqrt{1+x}}{x} + \tfrac{1}{2}\log\frac{\sqrt{1+x}-1}{\sqrt{1+x}+1},$$

while when $W<1$,

$$t=t_0(r)+\frac{2G}{(1-W^2)^{\frac{3}{2}}}\left\{H\left(\frac{2G}{Y(1-W^2)}\right)-H\left(\frac{2G}{r(1-W^2)}\right)\right\}, \tag{19}$$

where

$$H(x)=\frac{\sqrt{x-1}}{x}+\tan^{-1}\sqrt{x-1}.$$

$\left(\text{In this case we must always have } Y\leqslant\frac{2G}{1-W^2}\right)$.

Finally, if $W=1$,

$$t=t_0(r)+\frac{Y^{\frac{3}{2}}-r^{\frac{3}{2}}}{\sqrt{\frac{9}{2}G(r)}}. \tag{20}$$

We may conclude the appendix by giving the components of the Riemann-Christoffel tensor.

We find that

$$\left.\begin{aligned} R_{1212}=\frac{R_{1313}}{\sin^2\theta}&=-\frac{Y'^2}{W^2}\left(\frac{\lambda}{3}Y^2+\frac{G'}{Y'}-\frac{G}{Y}\right),\\ R_{2323}&=-Y^2\sin^2\theta\left(\frac{\lambda}{3}Y^2+\frac{2G}{Y}\right),\\ R_{0202}=\frac{R_{0303}}{\sin^2\theta}&=\frac{\lambda}{3}Y^2-\frac{G}{Y},\\ R_{0101}&=\frac{Y'^2}{W^2Y^2}\left(\frac{\lambda}{3}Y^2-\frac{G'}{Y'}+2\frac{G}{Y}\right). \end{aligned}\right\} \tag{21}$$

All other independent components vanish.

The invariant $B=R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ is given by

$$B=\frac{8}{3}\lambda^2+\frac{8}{3}\lambda\frac{G'}{Y^2Y'}+12\frac{G'^2}{Y^4Y'^2}-32\frac{GG'}{Y^5Y'}+48\frac{G^2}{Y^6}. \tag{22}$$

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INTERPRETATION OF ELECTRON DENSITIES FROM CORONA BRIGHTNESS

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Summary

The electron density in the corona as determined from observations of coronal brightness will be too high if an appreciable part of the coronal emission is not caused by scattering of electrons. The presence of Fraunhofer lines, and the diminution of polarization with ρ, the radial distance from the Sun, can both be explained by the existence of a non-electron source of light. It is found that the $\rho^{-2.5}$ term in Baumbach's empirical formula can be identified with the non-electron component of the coronal radiation.

The change of electron density with ρ has been tabulated after allowing for the non-electron component. The interpretation is in reasonable agreement with (*a*) the intensities of coronal Fraunhofer lines, (*b*) the diffraction theory of non-electron scattering, (*c*) the change of polarization with ρ, and (*d*) the change of polarization with orientation. The distribution of temperature with ρ is determined from the new electron densities.

1. *Introduction.*—The problem of electron density in the solar corona has been treated by M. Minnaert (**1**) and S. Baumbach (**2**), both of whom employ the scattering coefficient of electrons to derive electron densities from the intensity of coronal light. The values published by Baumbach are the ones in common use, because these were obtained from a careful consideration of all the coronal photometric data available. As the coronal details are quite variable there is no need to aim at higher accuracy than that represented by Baumbach's mean. However, Baumbach assumed that *all* coronal light was due to electron scattering, and if a substantial part of the light from the outer corona comes from another source, as suggested by Grotrian (**3**), then Baumbach's electron densities in the outer corona will be too high. In the present paper an allowance has been made for the non-electron source of coronal light, and the distribution of coronal light scattered by electrons alone has been deduced.

There are two observations to show that in the light from the corona there is a non-electron component which becomes of greater importance as the distance from the Sun increases. The high temperatures now accepted for the corona broaden the Fraunhofer lines in the light scattered by electrons until it is a continuous spectrum. Hence the Fraunhofer lines visible in the outer corona must be due to sunlight scattered or diffracted by some other medium. Let J_F be the intensity of the component that produces Fraunhofer lines, J_K the intensity of the continuous light scattered by electrons, and $J_S = J_F + J_K$. The intensities of the Fraunhofer lines were first used by Grotrian (**3**) to determine the relative intensity of J_F and J_K. In addition, the degree of polarization can be used to show that all the coronal light is not scattered by electrons. As the polarization in the outer corona is less than the theoretical value, it implies the existence of a non-polarized component, and we see good reason for identifying

this component with J_F. In principle, polarization measurements also provide a measurement of the ratio J_F to J_K.

In practice we prefer to use the Fraunhofer intensity measurements of J_F/J_K, since the measurements are less affected by extraneous light and the two sets of measurements that are available (**3**, **4**) are in reasonable agreement. Polarization measurements are so varied that the means cannot be used with safety, and these are used only as a check on the result obtained from the Fraunhofer intensities.

2. *Determination of J_K from Fraunhofer Intensities.*—The procedure for determining J_F/J_K from coronal Fraunhofer intensities has been described by Grotrian (**3**) and Allen (**4**), and depends simply on the relation

$$d_c/d_s = J_F/J_S = J_F/(J_F + J_K),$$

where d_c and d_s are the depths of corresponding Fraunhofer lines in the corona and the Sun respectively.

In Grotrian's work (**3**), an attempt was made to correct for extraneous light from the equipment and the Earth's atmosphere, but the corrections were regarded as unsafe beyond about $\rho = 2{\cdot}5$, where ρ is the projected radial distance from the Sun in terms of the Sun's radius. His values for J_K/J_S are given in column 3 of Table I.

Values of $J_K/J_S (= 1 - J_F/J_S)$ have also been given by Allen (**4**) and are reproduced in column 5 of Table I. No correction for extraneous light was made

TABLE I

Intensities of the components J_F and J_K in the coronal light and electron densities computed from J_K

(The intensity unit is 10^{-6} of the Sun's central intensity)

1	2	3	4	5	6	7	8	9
		Grotrian		Allen				
ρ	J_S	$\frac{J_K}{J_S}$	J_K Obs.	$\frac{J_K}{J_S}$	J_K Obs.	J_K Calc.	J_F Calc.	Electron densities from J_K
1·0	4·04	1·00	4·04	1·00	4·04	3·99	0·053	$4{\cdot}54 \times 10^8$
1·1	1·28	0·95	1·22	0·99	1·27	1·24	0·042	1·54
1·2	0·55	0·90	0·49	0·98	0·54	0·52	0·033	0·685
1·3	0·284	0·85	0·24	0·96	0·273	0·256	0·028	0·366
1·4	0·166	0·80	0·133	0·94	0·156	0·143	0·023	0·220
1·6	0·0704	0·70	0·050	0·87	0·062	0·054	0·0164	0·093
1·8	0·0356	0·62	0·0220	0·78	0·028	0·0234	0·0122	0·0457
2·0	0·0205	0·53	0·0109	0·68	0·0140	0·0111	0·0094	0·0242
2·2	0·0131	0·44	0·0058	0·57	0·0075	0·0058	0·0074	0·0138
2·4	0·00906	0·35	0·0032	0·46	0·0042	0·0031	0·0060	0·0082
2·6	0·00665	0·27	0·0018	0·35	0·0023	0·0018	0·0048	0·0050
2·8	0·00512					0·0010	0·0041	0·0032
3·0	0·00407					0·00065	0·0034	0·00213
3·5	0·00257					0·00022	0·00235	0·00085
4·0	0·00175					0·00009	0·00166	0·00038
5·0	0·000968					0·00002	0·00095	0·00010
6·0	0·000608						0·00061	0·000033
8·0	0·000294						0·00029	0·000006
10·0	0·000168						0·00017	0·0000016

in this case, but in the outer corona the ratio of the sunlight from the sky to the instrumentally scattered light from the corona is likely to be of the same order as J_F/J_K, and hence the correction should be small. In any case, the extraneous light at $\rho=2{\cdot}6$ was rather small compared with the coronal light.

We use Baumbach's data (**2**) for the mean distribution of J_S with ρ. His values are given in column 2 of Table I in units which equal 10^{-6} of the Sun's central intensity I_0. When multiplied by the two determinations of J_K/J_S, they give the J_K distributions of columns 4 and 6. It will be recalled that Baumbach's coronal intensities are fitted to the formula

$$J_S(\rho)=0{\cdot}0532\rho^{-2{\cdot}5}+1{\cdot}425\rho^{-7}+2{\cdot}565\rho^{-17}, \qquad (1)$$

where the unit is again 10^{-6} of the Sun's central intensity. As the outward decrease of intensity is more rapid for J_K than J_S, it is interesting to omit the $\rho^{-2{\cdot}5}$ term. The ρ^{-7} and ρ^{-17} terms are added in column 7 of Table I and can be seen at once to fall close to the J_K values determined by Grotrian and Allen. Thus, in the range $\rho=1{\cdot}2$ to $2{\cdot}6$, the intensity J_K which is scattered by electrons follows the formula

$$J_K(\rho)=1{\cdot}425\rho^{-7}+2{\cdot}565\rho^{-17} \qquad (2)$$

and the non-electron component J_F follows the formula

$$J_F(\rho)=0{\cdot}0532\rho^{-2{\cdot}5} \qquad (3)$$

This rule, derived from Fraunhofer intensity measurements, is now applied to other data and found to agree admirably.

3. *Polarization.*—The polarization of the corona has been treated theoretically by A. Schuster (**5**), M. Minnaert (**1**), and S. Baumbach (**6**), of whom Baumbach has computed the polarization as a function of ρ for all three terms of his formula (his Table III). The polarization associated with the ρ^{-7} and ρ^{-17} terms combined is shown in Fig. 1. This diagram also shows the resulting curves when the $\rho^{-2{\cdot}5}$ term is added, either polarized or unpolarized.

This same diagram gives the polarization measurements by Young (**7**), Fessenkoff (**8**), Dufay and Grouiller (**9**), Johnson (**10**), and Cohn (**11**), which were discussed in Baumbach's paper, and those which I have made more recently (**12**). There is great variety among the polarization measurements but the tendency is to follow the curve representing $\rho^{-2{\cdot}5}$ term unpolarized. Some explanation of the variety found in polarization measurements is given in Section 5.

Baumbach (**6**) has himself explained the decrease of polarization with ρ as being due mainly to the influence of non-polarized extraneous light from the sky. His estimate of sky-intensity $2A(R)$ at about $\rho=2$ is actually greater than the $\rho^{-2{\cdot}5}$ term in his equation. It seems to me that this estimate is much too high. The brightness of the sky in the neighbourhood of the Sun during eclipses has been estimated by Parkhurst (Farnsworth (**13**)), Kunz and Stebbins (**14**) and King and Harwood (**15**), who have determined the light from a stated area of sky in terms of the total coronal light. Taking the corona brightness as $1{\cdot}0\times10^{-6}$ that of the Sun, and the mean/central solar intensity $\bar{I}/I_0$ as $0{\cdot}77$, we obtain the values of sky brightness in terms of the Sun's centre given in Table II. I have included also observations made from over-exposed polarization plates at the 1940 eclipse. The average sky brightness $0{\cdot}0014\times10^{-6}$ is about ten times smaller than that used by Baumbach near $\rho=2$. Since sunlight does not reach

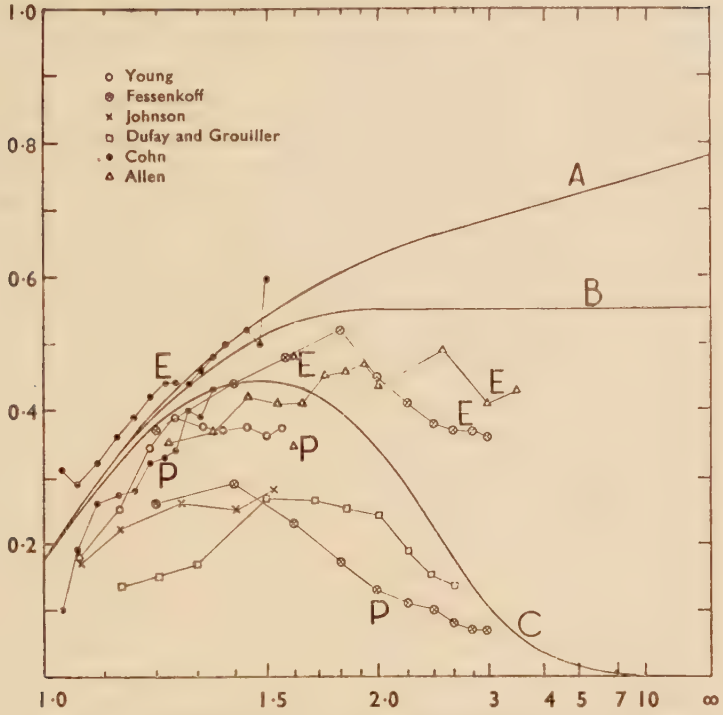

FIG. 1.—*Polarization as a function of radial distance from the Sun.*
A.—ρ^{-7} *and* ρ^{-17} *terms combined, both polarized.*
B.—$\rho^{-2\cdot5}$, ρ^{-7} *and* ρ^{-17} *terms all polarized.*
C.—$\rho^{-2\cdot5}$ *term not polarized,* ρ^{-7} *and* ρ^{-17} *terms polarized.*
E.—Equator.
P.—Pole.
Ordinates : degree of polarization.

the sky directly there is no reason why the sky brightness should increase with decrease of ρ as assumed by Baumbach.

TABLE II

Brightness of the sky in the neighbourhood of the Sun during an eclipse

Parkhurst (Farnsworth (**13**))	Neutral, Visual	$0\cdot0011\times10^{-6}$	centre of Sun
Kunz and Stebbins (**14**) (1918)	Violet, Photoelectric	$0\cdot0016\times10^{-6}$	,, ,,
King and Harwood (**15**) (1925)	Violet, Photographic	$0\cdot0020\times10^{-6}$	,, ,,
Allen (1940)	Blue, Photographic	$0\cdot0014\times10^{-6}$	,, ,,
Allen (1940)	Red, Photographic	$0\cdot0009\times10^{-6}$	,, ,,

4. *Electron Densities.*—Electron densities were derived by Baumbach in his first paper (**2**), and were, of course, based on all three terms of his formula. Fortunately, his theory treats the three terms individually, and the electron density resulting from (2) is readily found to be:

$$N(\rho)=10^{8}(1\cdot55\rho^{-6}+2\cdot99\rho^{-16}).$$

The number of electrons per cubic centimetre from this formula are given in column 9 of Table I.

It will be seen that the corona has dropped to about 10^{-3} of its surface density at $\rho=2\cdot6$ instead of at $\rho=4\cdot5$ as obtained from the three-term formula. This

difference may have a great significance when applying electron densities and magnetic fields together to problems connected with the emission of solar radio-noise of rather longer wave-length.

One interesting application of electron densities has been the calculation by Alfvén of coronal temperatures (**16**) from the equation of gravitational equilibrium. Putting $T = 1{\cdot}16 \times 10^7 E/E_0$ in Alfvén's terminology, his temperature distribution is given in Fig. 2. This is taken from Fig. 1 of Alfvén's paper and neglects the force due to inhomogeneous magnetic fields, which, however, does not greatly alter the character of the curve. Alfvén's theory can readily be applied with the first term ($\rho^{-1{\cdot}5}$ in electron density) omitted and yields the temperature variation also shown in Fig. 2. The difference might be of some importance for coronal interpretation.

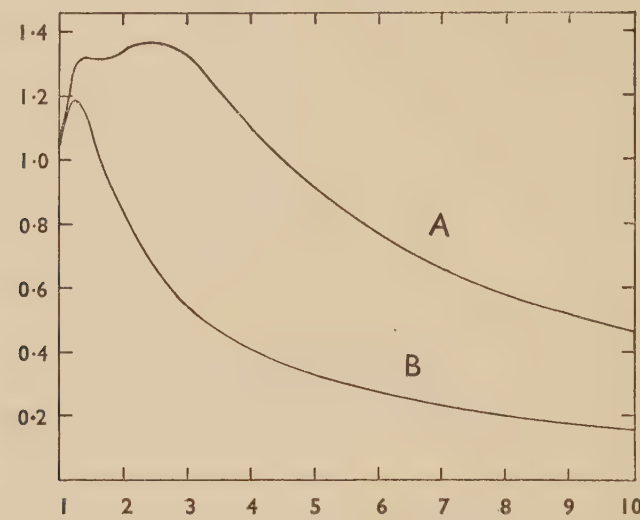

Fig. 2.—*Relation between temperature and radial distance from Sun.*
A.—Electron densities determined from J_S.
B.—Electron densities determined from J_K, i. e. first term omitted.
Ordinates : temperature $\times 10^{-6}$.

5. *The J_F Component.*—The author (**4**) has identified the J_F component of the corona with diffraction of sunlight by interplanetary particles, and has found best agreement with observations if one takes the radius of the diffracting particles as 10^{-3} cm., the number of particles per c.c. at the distance of the Earth N_E as 3×10^{-15}, and the relation between number density and distance from the Sun r as $N(r) = N_E(D/r)$, where D is the Sun-Earth distance. Using these values in Table VI of my paper (**4**) one finds $J_F/\bar{I}$ (where $\bar{I}$ is the mean brightness of the Sun's surface) almost independent of wave-length and having the values given in Table III. Using $\bar{I}/I_0 = 0{\cdot}77$, we obtain J_F/I_0 for comparison with the corresponding values obtained by putting J_F = the $\rho^{-2{\cdot}5}$ term. The agreement

Table III

ρ	$J_F/\bar{I}$	J_F/I_0	J_F/I_0
	Calculated from diffracted light		Calculated from J_F = the $\rho^{-2{\cdot}5}$ term
1·5	$0{\cdot}0258 \times 10^{-6}$	$0{\cdot}0198 \times 10^{-6}$	$0{\cdot}0194 \times 10^{-6}$
2·0	$0{\cdot}0132 \times 10^{-6}$	$0{\cdot}0102 \times 10^{-6}$	$0{\cdot}0094 \times 10^{-6}$
2·5	$0{\cdot}0066 \times 10^{-6}$	$0{\cdot}0051 \times 10^{-6}$	$0{\cdot}0054 \times 10^{-6}$

could hardly be better, and in particular it should be noticed that the change of J_F/I_0 with ρ, as calculated from diffraction theory, follows very nearly the $\rho^{-2.5}$ power law.

If the J_F component is produced by diffraction of zodiacal light particles the amount of light diffracted should be symmetrical around the Sun for distances of a few degrees, because the zodiacal light extends for several degrees on either side of the ecliptic and the Earth–Sun line lies in the ecliptic plane. Grotrian's observations (**3**) support this symmetry to some extent by showing that J_F is approximately equal on the east and west side of the Sun. A much more sensitive test would be to measure J_F/J_K at the pole and equator at sunspot minimum, when J_K will be greater at the equator than at the pole. If J_F is a constant function of ρ, any variation of J_K from one eclipse to another would affect the ratio J_F/J_K, and this can account for the differences between Grotrian's and Allen's observations.

The constancy of J_F will cause a change of polarization with orientation, especially at sunspot minimum, when the polarization should be less at the pole than at the equator. Very considerable differences between polar and equatorial polarization have been observed (**8**, **11**) and always in the sense that the equatorial polarization is greater. In Fig. 1 it is seen that there is a tendency for the equatorial polarization to be greater, and the polar polarization to be less, than the curve based on J_F non-polarized. Thus, in spite of the scatter, the polarization measurements give reasonable agreement with our interpretation of J_F and J_K. However, we can give no explanation of the difference between polar and equatorial polarization that Cohn has observed between $\rho = 1{\cdot}0$ and $1{\cdot}3$.

One can compute the equatorial and polar polarization if the ratios of the equatorial and polar intensities to the mean, viz. J_{KE}/J_K and J_{KP}/J_K, are known. For $\rho = 1{\cdot}6$ the photometry of the 1940 eclipse gave:

$$\frac{J_F+J_{KE}}{J_F+J_K} = 1{\cdot}66 \quad \text{and} \quad \frac{J_F+J_{KP}}{J_F+J_K} = 0{\cdot}38.$$

Using $J_F = 0{\cdot}30 J_K$ from columns 7 and 8 of Table I, we get:

$$\frac{J_{KE}}{J_F+J_{KE}} = 0{\cdot}86 \quad \text{and} \frac{J_{KP}}{J_F+J_{KP}} = 0{\cdot}40.$$

The equatorial and polar polarizations are obtained by multiplication of these values by 57 per cent, the polarization of J_K alone at $\rho = 1{\cdot}6$ (from Fig. 1), giving 49 per cent equatorial and 23 per cent polar. The percentage polarizations measured at $\rho = 1{\cdot}6$ (mean from $\rho = 1{\cdot}45$ to $1{\cdot}75$) are:

E 47·3	W 49·0	Mean equatorial	48·2
N 35·6	S 32·7	Mean polar	34·6

The equatorial agreement is good and the polar agreement reasonable when one considers that differences between rather inaccurately determined quantities are involved. However, later consideration has led to the view that the 1940 polarization measurements in the fainter parts of the corona are somewhat over-corrected and too high. If this is so, the polarization will fall closer to our interpretation.

Conclusion.—It would indeed be surprising if a term used in a power equation adjusted to fit the curve connecting J_S with ρ were to have a physically separate meaning and also have exactly the right coefficient. We have found, nevertheless,

that by identifying the $\rho^{-2.5}$ term with J_F, which we ascribe to diffraction of interplanetary particles, we have been able to account for:

(*a*) the intensity of coronal Fraunhofer lines,
(*b*) the relation between J_F and ρ in terms of the diffraction theory,
(*c*) the change of polarization with distance from the Sun, and
(*d*) the change of polarization with orientation.

The greatest disagreement comes from my own polarization measurements, which are somewhat higher than required, and this cannot be entirely explained by the fact that all the more distant measurements were equatorial. It is thought now that the measurements themselves are rather high, being over-corrected; however, the measurements as they stand suggest that J_F is somewhat less than $0{\cdot}0532\ \rho^{-2.5}$.

Another departure that would help the agreement a little would be to allow that J_K does not cut off in the outer corona as severely as given by the ρ^{-7} and ρ^{-17} terms. I think the best representation of the facts is made by assuming that the corona proper (for J_F is not really coronal light at all) follows the ρ^{-7} and ρ^{-17} terms in general, but that those fine straight streamers that can sometimes be traced to $\rho = 5$ should be regarded in the outer corona as additional to the regular terms.

The polarization of individual streamers at right-angles to the line of sight at $\rho = 5$ would reach 95 per cent (Minnaert's (1) Table II), and hence the outer polarization would depend on whether such streamers were present. Polarization was, in fact, detected in 1940 as far as $\rho = 4{\cdot}5$, by noting the complete disappearance of streamers with the rotation of the polariod.

The author's thanks are due to Dr R. v. d. R. Woolley for suggesting that this analysis be carried out.

Addendum, 1948 *March :* In a paper received during the course of publication H. C. van de Hulst (*Ap. J.*, **105**, 471, 1947) has used similar arguments to determine coronal electron densities, and obtains nearly the same result. He also gives strong support for the argument that the non-electron corona is caused by diffraction of zodiacal light particles.

Commonwealth Observatory,
Mount Stromlo, Canberra :
1947 *May* 20.

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AN ANALYSIS OF SOLAR ATMOSPHERIC STRUCTURE

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Summary

The problem of the analysis of observations of solar limb darkening is examined and it is concluded that deficiencies in observational data always compel the adoption of an auxiliary physical hypothesis. A possible hypothesis is discussed and its consequences shown to be in agreement with observations of limb darkening and Fraunhofer line variation across the solar disk. The line strength variations to be expected in the infra-red on the basis of the foregoing analysis are computed. It is expected that line strengthening to the limb will be a relatively common phenomenon in this spectral region.

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1. *The Analysis of the Continuous Solar Spectrum.*—The Sun is the only star which presents to observation a visible disk and, therefore, is the only star (with the doubtful exception of some eclipsing binaries) for which, at the moment, there is any hope of a determination of the march of physical conditions with increasing depth below the surface. Numerous analyses of the observed behaviour of the continuous spectrum and of the variation of line intensities across the disk have been given by various authors * which have yielded results showing a very considerable degree of variation, one from another.

The problem of the analysis of the centre-to-limb variation of the continuous spectrum is the problem of the solution of the integral equation

$$I_\lambda(0,\theta)=\int_0^\infty B_\lambda(\tau)\sec\theta\, e^{-\tau\sec\theta}d\tau, \tag{1}$$

where $I_\lambda(0,\theta)$ represents the intensity emergent at the solar surface at an angle θ to the normal in wave-length λ, and $B_\lambda(\tau)$ represents the temperature radiation in the same wave-length at optical depth τ. The problem is, given $I_\lambda(0,\theta)$ at any wave-length, to deduce $B_\lambda(\tau)$.

This problem presents a number of difficulties of considerable subtlety, both of a mathematical and an observational kind, and it will be convenient to discuss them under these headings. In order to discuss the mathematical aspects of the problem, unmixed with physical considerations, let us suppose that we have in our possession data for $I_\lambda(0,\theta)$ of perfect accuracy extending throughout the range from $\theta=0°$ to $\theta=90°$. The solution to the problem is then quite determinate, and can be obtained in a number of ways, of which, perhaps, the method of expansion in series is the most illuminating. As is well known †, if we write

$$B_\lambda(\tau)=a_0+a_1\tau+a_2\tau^2+\ \ldots\ +a_n\tau^n+\ \ldots, \tag{2}$$

then

$$I_\lambda(0,\theta)=a_0+a_1\cos\theta+2a_2\cos^2\theta+\ \ldots\ +n!a_n\cos^n\theta+\ \ldots. \tag{3}$$

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* H. H. Plaskett, *M.N.*, **96**, 402, 1936; *M.N.*, **101**, 3, 1941; H. Raudenbusch, *A.N.*, **266**, 301, 1938; I. W. Busbridge, *M.N.*, **101**, 26, 1941; M. G. Adam, *M.N.*, **98**, 112, 1938 (Paper I) and *M.N.*, **100**, 595, 1940 (Paper II); D. Chalonge and M. V. Kourganoff, *Ann. d'Astrophys.*, **9**, 69, 1946.

† M. G. J. Minnaert, *Z. f. Ap.*, **12**, 313, 1936.

If our knowledge of $I_\lambda(0,\theta)$ is perfect and we express it as a function of $\cos\theta$, and if, as we may expect to be the case, this function is expressible as a power series in $\cos\theta$, we may determine the coefficients a_n from the geometrical properties of the curve near $\cos\theta=0$. The intercept, gradient, first, second and higher curvatures yield respectively multiples of a_0, a_1, a_2, a_3 and a_n, etc. This solution is completely ideal and a discussion along these lines has already been given for the case of absorption line formation.* The important point is that there is a most intimate relation between the geometrical properties of the curve representing $I_\lambda(0,\theta)$ near $\cos\theta=0$ and the curve representing $B_\lambda(\tau)$ near $\tau=0$.

The solution has no practical value because observations of limb darkening having sufficient precision have not been pushed right to the limb of the Sun. Existing observations even of the integrated light of the solar crescent during eclipses† do not help us, since it can be shown that these are very insensitive to the assumed law of darkening.‡ We must, perforce, remain for the present in ignorance of just that range of observed values which is of the greatest importance in determining the form of the function $B_\lambda(\tau)$, and are, in practice, limited to a range of observational data which does not extend nearer to the limb than the zone for which $\cos\theta=0{\cdot}25$ roughly. We must consider whether this limitation on our knowledge constitutes a serious obstacle to the determination of $B_\lambda(\tau)$ or not. Chalonge and Kourganoff§ consider that it does not, and, in support of this contention have shown that, if the series in equations (2) and (3) are limited to three terms, a form giving an excellent representation of the observations, then the results for $B_\lambda(\tau)$ obtained by this method are in good agreement with those obtained if the functional form suggested by Miss Busbridge‖ is used instead. A closer consideration shows that this contention is not valid, the agreement being due apparently to certain underlying geometrical similarities in the forms of the two extrapolations beyond $\cos\theta=0{\cdot}25$.

Let us consider, for example, the case of the observations of Raudenbusch at λ 5960 A., which Chalonge and Kourganoff represent by the function

$$\phi(\theta)=\frac{I_\lambda(0,\theta)}{I_\lambda(0,0)}=0{\cdot}3959+0{\cdot}7271\cos\theta-0{\cdot}1244\cos^2\theta. \qquad (4)$$

This fits the observations down to $\cos\theta=0{\cdot}25$ and extrapolates the observed curve to 0·3959 at the extreme limb. It yields for $B_\lambda(\tau)$ the function

$$B_1(\tau)=0{\cdot}3959+0{\cdot}7271\tau-0{\cdot}0622\tau^2. \qquad (5)$$

Now let us suppose that, quite arbitrarily, we retain all the observed values for the region for which $\cos\theta>0{\cdot}25$, but extrapolate into the small values of $\cos\theta$ by drawing a smooth curve through arbitrarily chosen points, say 0·1000 at $\cos\theta=0$ and 0·5220 at $\cos\theta=0{\cdot}2$. We find that the expression

$$\phi(\theta)=0{\cdot}1000+3{\cdot}7485\cos\theta-11{\cdot}7175\cos^2\theta +20{\cdot}9729\cos^3\theta-17{\cdot}9896\cos^4\theta+5{\cdot}8854\cos^5\theta \qquad (6)$$

fits this curve, and gives for $B_\lambda(\tau)$ the expression

$$B_2(\tau)=0{\cdot}1000+3{\cdot}7485\tau-5{\cdot}8588\tau^2+3{\cdot}4955\tau^3-0{\cdot}7496\tau^4+0{\cdot}04905\tau^5. \qquad (7)$$

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* D. S. Evans, *M.N.*, **101**, 21, 1941.
† A. J. Wesselink, *et al.*, *B.A.N.*, **9**, 81, 1940. (See also Z. Kopal, *Ap. J.*, **104**, 60, 1946.)
‡ D. S. Evans, *Observatory*, **67**, 138, 1947.
§ *Loc. cit.*
‖ *Loc. cit.*

These two curves are shown in Fig. 1, and are markedly discrepant, in spite of the fact that, over the observed range, they both give the same darkening to the limb. We can, in fact, as we might have seen from mathematical considerations, choose quite arbitrarily that part of the darkening curve not actually observed, and still obtain a result for $B_\lambda(\tau)$ in agreement with observation. If our knowledge of $I_\lambda(0, \theta)$ is limited to a certain range of $\cos\theta$, then equation (1), regarded purely as a mathematical problem, possesses an almost infinite variety of solutions for $B_\lambda(\tau)$, the differences between which are not necessarily confined, as Fig. 1 shows, to small values of τ. It should however be noted that these differences do become less and less important in a ratio of the order of $e^{-\tau}$ with increasing optical depth, and, for this reason, it would not be valid to criticize Chalonge and Kourganoff's result (equation (5)), solely on the ground that their $B_\lambda(\tau)$ becomes negative at great depths.

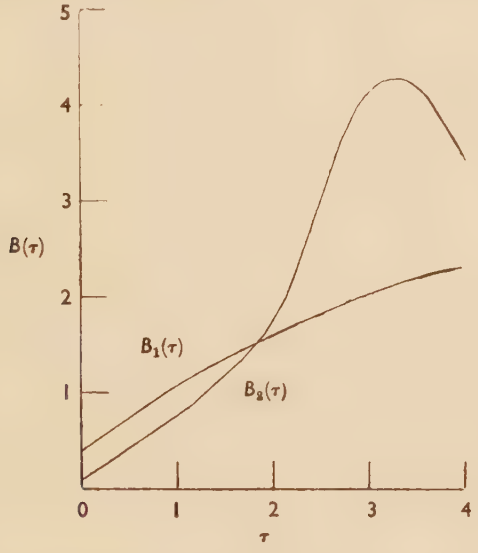

FIG. 1.

It will be clear that their extrapolation of the darkening curve to the extreme limb on the basis of a quadratic function of $\cos\theta$ constitutes not only a mathematical convenience but also an important physical assumption which largely determines the character of the solution obtained. In all the analyses of limb darkening which have been carried out, it has been necessary to make, either implicitly or explicitly, some auxiliary physical assumption to compensate for the lack of observational data in the neighbourhood of the extreme limb. In the work of Plaskett and of Raudenbusch the assumption seems to have been that the $B_\lambda(\tau)$ resulting from the analysis must be as close as possible to that which would be predicted by radiative equilibrium considerations. The assumptions of Miss Busbridge and of Barbier* are mathematical in form but physical in reality, since they prejudge the character of the darkening law at the extreme limb.

An additional complication is the possibility suggested by Redman† that the solar surface is rough, and which would entail either a complete recasting of the theory epitomized in equation (1) or the replacement of the apparent

* *Ann. d'Astrophys.*, **9**, 173, 1946.
† *M.N.*, **103**, 173, 1943.

values of $\cos\theta$ near the solar limb by a related series of values which do not fall to zero at the apparent limb.

Observations of solar limb darkening made on the unobscured disk and, in consequence, possibly subject to effects of atmospheric scattering and scintillation, are those of Abbott *, of Moll, Burger and van der Bilt †, and of Raudenbusch.‡ The first and last were obtained with apparatus involving one or more reflections of the incoming beam at silvered surfaces, and must be affected to an unknown extent by scattering at surface imperfections. It is worth noting that observations of Abbott at $\lambda 4265$ and of Raudenbusch at $\lambda 4260$ differ by sometimes as much as 3 per cent, although both sets of results are given to four places of decimals. There is, in addition, no guarantee that limb darkening is strictly constant with time.

After this preliminary discussion we may address ourselves to the problem of the available observations of centre-to-limb contrast, taking as rules of procedure:

(1) That in order to effect an analysis an auxiliary physical assumption must be made.

(2) That any solution which fits the observations within the limits of error must be accepted for further discussion, and that it is pointless to strain after any greater closeness of representation.

2. *Choice of an Auxiliary Physical Hypothesis.*—We must now consider what physical assumption ought to be made, and must compare its merits with those of possible alternatives. The basic assumption of radiative equilibrium is that the only means by which energy is transported through the solar atmosphere is by radiation, and hence, that stratification in plane parallel, stationary layers must obtain. It is now clear that this hypothesis cannot be strictly true. It is not consistent with the observed centre-to-limb darkening, and Plaskett's pioneer analysis of the continuous spectrum did suggest that at least a certain degree of convection must be postulated. This view receives strong support from the observed fact of solar granulation, which, it has been suggested, must be regarded as due to convection in Bénard cells.§ If we admit the existence of a convective zone, the next question is that of its extent. It is now generally accepted that the solar atmosphere consists mainly of hydrogen, and Eddington ‖, in an important paper based on this hypothesis, gave reasons for expecting large-scale convection due to the breakdown of hydrogen ionization equilibrium. Since the intensity of radiation in all frequencies beyond the Lyman limit, and hence the energy, available for the ionization of hydrogen, is rather small, it is conceivable that a downward-moving mass of unionized hydrogen might reach a considerable depth before becoming dissociated, even if its velocity were relatively small. Eddington concluded that a convection zone based on this breakdown of hydrogen ionization equilibrium might exist between the levels at which the equilibrium ionization of hydrogen would be 5 per cent and 95 per cent, and that the process would be self-maintaining. He himself pointed out that the

* *Ann. Ast. Obs. Smithsonian Inst.*, **3**, 157, 1913; **4**, 221, 1922. (The results used are as quoted by Chalonge and Kourganoff.)

† *B.A.N.*, **3**, 83, 1925. These results are stated by Chalonge and Kourganoff to be preliminary only.

‡ *Loc. cit.*

§ See, e. g. H. H. Plaskett, *loc. cit.*; J. Wasiutyński, *Astrophys. Norvegica*, **4**, 1946, Figs. 16 and 17.

‖ *M.N.*, **101**, 177, 1941.

effects of collisional ionization which he had neglected might vitiate this conclusion, and in a later paper Schwarzschild * suggested that in fact collisional ionization would restore the hydrogen ionization equilibrium. However, it may be noted that Schwarzschild adopted as temperature limits the values given by the radiative equilibrium law

$$T = T_0(1 + \tfrac{3}{2}\tau)^{\frac{1}{4}}. \tag{8}$$

Collisional ionization depends on the presence of electrons of sufficient energy and in a large convecting mass, such as a granule, which is not entirely at the mercy of ambient conditions, it seems likely that electrons of an energy sufficient to effect ionization by collision will not be present in appreciable numbers until photoionization has become appreciable. Schwarzschild does in fact seem to have shown that for a mass of hydrogen in contact with the lower boundary of Eddington's convection layer, collisional ionization will be appreciable, but for a neutral mass moving inwards this may not be true.

In a later paper, Eddington †, having accepted Schwarzschild's conclusions, invoked the mechanism of adiabatic convection in maintaining a position not far removed from his original one, and followed Unsöld ‡ in postulating an extensive hydrogen convection zone. On the basis of equation (1) Eddington's limits for the hydrogen convection zone, which he placed at 9000 deg. K. and 27,000 deg. K., would lie at optical depths 3·8 and 375, while Unsöld's analysis gave depths of 2·0 and 30.

In formulating a model to describe this situation it cannot be asserted that the limits of the convection zone would correspond to those deduced in the arguments sketched above, and it was considered that a preliminary rough description might be secured by postulating a constant value of $B(\tau)$ in the convection zone, considered to extend from some small optical depth down to great (effectively infinite) optical depths. Above the convection layer the temperature presumably increases with depth and a rough model will represent $B(\tau)$ as increasing linearly in this region. In the discussion it was decided not to impose a continuity condition at the upper surface of the convection layer, since it might happen that, at the transition, the temperature would increase rapidly, approximating to a continuous jump. If in fact the best model should be a continuous one, this, it was hoped, would emerge from the discussion.

The bulk of the work had actually been completed when there came into our possession photostats of two papers written in criticism of Eddington's first contribution, but clearly also in ignorance of his later modified views. The first, by Unsöld §, criticizes particularly Eddington's original result that there would be a marked departure from ionization equilibrium in the convection zone. Clearly, the ideal way to discuss a non-equilibrium problem of this kind is not by the use of general thermodynamic parameters which describe the overall physical state of material, but rather to consider the separate action of each microscopic atomic process, and the rôle which it plays in tending to bring about a state of equilibrium. This ideal is exceedingly difficult to attain, and Unsöld does not succeed in doing so. His work seems to be open to criticism on two grounds : in the first place he applies Saha's ionization law to determine the degree

* *M.N.*, **102**, 152, 1942.
† *M.N.*, **102**, 154, 1942.
‡ *Z. f. Ap.*, **1**, 138, 1930.
§ *Z. f. Ap.*, **21**, 307, 1942.

of dissociation of hydrogen, which, since this is a "thermodynamic" formula certainly involves a logical and perhaps even a serious physical error. In the second place he explicitly restricts his discussion to aggregations of material of small optical thickness, which seems far from correct if the discussion is intended to apply to granulation structure.

The second paper, by Biermann*, seems to approach more closely to the ideal of a microscopic discussion. He also concludes with Unsöld that deviations from hydrogen ionization equilibrium are small, but in many important respects his results and those of Unsöld are at variance, and it seems impossible to come to a decision as to which is nearer the truth without carrying out the whole theoretical discussion afresh. It is worth noting that Biermann gives curves showing the march of temperature with depth on two alternative assumptions : one, that energy transport in the convective zone is entirely due to convection; the other that it is due entirely to radiative processes. Somewhat surprisingly, the latter alternative yields a result similar, especially in the sudden increase of $B(\tau)$ predicted for the upper boundary of the convection zone, to that finally deduced in the present paper. It seems worth while therefore to reproduce Biermann's figure here (Fig. 2).

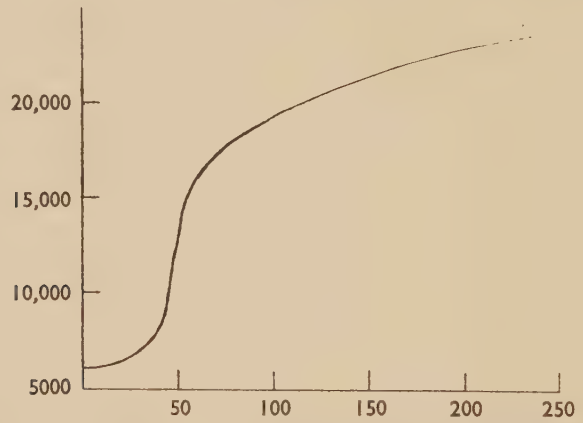

FIG. 2.—*March of temperature with depth in the solar atmosphere (after Biermann).*
Ordinates : Temperature (deg. K.).
Abscissae : Depth in km.

These theoretical discussions leave the matter in a state of some obscurity, although it is generally agreed that a convection layer extending to great depths exists. In view of the sequel it is worth noting that Biermann's layer extends from optical depth 1·1 to great depths. The point at issue is whether or not there is a marked departure from hydrogen ionization equilibrium. The comment may be offered that the chief danger in these theoretical discussions seems to lie in circular argument : if, for example, an author sets up a series of postulates in which he assumes the Saha law to be true, there does seem to be a danger that he will thereby restrict his discussion to a physical framework within which he will find the Saha law approximately valid. If, on the other hand, he were to assume that some other thermodynamic law were approximately fulfilled, he would work in a quite different framework in which the relative importance assigned to the different physical processes involved might be quite different,

Z. f. Ap.*, **21, 320, 1942.

and he might conclude that marked deviations from equilibrium ionization existed.*

From our point of view, the important question is whether or not the presence of the convection layer will involve a region of small temperature gradient. If the works cited have not been misunderstood, it seems that in some cases the answer to this question would be affirmative, in others negative. This leaves us free to make what assumptions we please, within limits, and, since the consequences of the second alternative have been discussed frequently, we shall adopt the first, namely, that convection does involve a "plateau" on the $B(\tau)$ curve, and compare the consequences with observation.

3. *Mathematical Formulation.*—As has already been indicated, the mathematical form which we employ is very simple, and it must be emphasized that it is regarded as an approximation, the real distribution of $B(\tau)$ being considered to be a smoothed version of our crude discontinuous model. We assume that the convection layer starts at some optical depth τ_0 and extends to infinite optical depth, and that its temperature radiation is constant, having $B(\tau)=\gamma$. In the region above the convection layer we use the approximation $B(\tau)=\alpha+\beta\tau$. Continuity of $B(\tau)$ is not assumed, but we do assume that $B(\tau)$ nowhere decreases with increasing depth, so that this proviso does form part of our auxiliary physical assumption.

We take

$$\left.\begin{aligned} B(\tau)&=\alpha+\beta\tau \qquad (\tau<\tau_0)\\ B(\tau)&=\gamma \qquad\quad (\tau>\tau_0)\end{aligned}\right\} \tag{9}$$

and find

$$\frac{I(0,\theta)}{I(0,0)}=X=\frac{(\alpha+\beta\cos\theta)(1-e^{-\tau_0\sec\theta})+e^{-\tau_0\sec\theta}(\gamma-\beta\tau_0)}{(\alpha+\beta)(1-e^{-\tau_0})+e^{-\tau_0}(\gamma-\beta\tau_0)}. \tag{10}$$

We denote the denominator in (10) by D, and write

$$\alpha'=\alpha/D \qquad \beta'=\beta/D \qquad \gamma'=\gamma/D. \tag{11}$$

We write

$$\mu'=\gamma'-\beta'\tau_0-\alpha' \tag{12}$$

which gives

$$\mu'=\beta'+e^{\tau_0}(1-\alpha'-\beta'), \tag{13}$$

so that (10) becomes

$$X=\alpha'+\beta'\cos\theta+e^{-\tau_0\sec\theta}(\mu'-\beta'\cos\theta), \tag{14}$$

and this we may put in the form

$$\alpha'.p+\beta'.q=X-P, \tag{15}$$

where

$$P=1-p=e^{-\tau_0(\sec\theta-1)}: \quad q=\cos\theta(1-e^{-\tau_0\sec\theta})+e^{-\tau_0\sec\theta}-e^{-\tau_0(\sec\theta-1)}. \tag{16}$$

4. *Numerical Solution.*—At each wave-length the observed values of X as defined in equation (10) for $\cos\theta=0{\cdot}25$, $0{\cdot}3$, $0{\cdot}4$, $0{\cdot}5$, $0{\cdot}75$ and $1{\cdot}00$ were taken. Some value of τ_0 was assumed, and the values of the coefficients p, q and P for the given cosines computed. A solution for α' and β' was then obtained by the method of least squares applied to equation (15), and the parameters μ' and γ'

* Criticism of this kind seems applicable to a paper by Rudkjöbing, *Z. f. Ap.*, **21**, 254, 1942, "*Über Konvektion in Sternatmosphären*", in which considerable reliance is placed on equilibrium formulae.

obtained from equations (12) and (13). The arrangement of the work is such that X automatically equals unity at $\cos\theta = 1{\cdot}00$, so that this provides a check, but the agreement of the computed and observed values at the other five points enables a measure of the goodness of fit of the computed solution to be obtained. The simplest method seems to be to give the modulus $|\Delta_{max}|$ of the maximum difference between observation and computation, together with the value $\sqrt{(\Sigma\Delta^2/n)}$ of the root mean square difference at the five points.

This procedure was adopted in the first instance because of the difficulty of finding τ_0 as well as α' and β' by any simple method, but the curious result emerged that, at all wave-lengths, it was possible to find solutions agreeing with observation over a wide range of values of τ_0. In fact, what limits the range of possible values of τ_0 is not a falling off in accuracy of the fitted curves until the discrepancies exceed the observational errors, but the condition imposed above that no solution in which the temperature decreases inwards can be admitted on physical grounds. The range is limited by the fact that for small values of τ_0, β' becomes zero, while for larger values μ' (which, as can be seen from equation (12), gives the height of the discontinuity at τ_0) becomes zero.

It was thus necessary in the case of each set of observational results to solve a series of equations for different assumed values of τ_0, and, in view of the considerable labour involved, it was a convenience to adhere to standard sets of values of $\cos\theta$ and of τ_0, for which the sums of squares and products of the coefficients p, q and P defined in equation (16) could be calculated in advance. It may be noted that, from the purely mathematical point of view, the present method accounts for the rather small curvature in the contrast curve by means of the exponential product in equation (14), whereas Chalonge and Kourganoff achieve this by the inclusion of a term in $\cos^2\theta$.

The following table shows the data analysed by this method :

TABLE I

Wave-lengths of observations analysed

Authority		
Moll, Burger and van der Bilt	Raudenbusch	Abbott
4500	4260	4265
5000	5060	5062
5500	5960	10080
6000	6700	12250
6500		16650
7000		20970
8000		
9000		
11000		
15000		

It should be explained that the original intention was to base the present study on the observations of Moll, Burger and van der Bilt, and of Raudenbusch. The selected wave-lengths from Abbott's observations were included later to give additional information on specific points which emerged from the discussion. Approximately 150 sets of least squares equations were solved in the analysis.

The results can be summarized as follows : The maximum value of $|\Delta_{max}|$ over all 150 cases was 2 per cent, which was attained in one instance : the maximum value of $\sqrt{(\Sigma\Delta^2/n)}$ was 1 per cent, which was attained in only 4 cases out of 150. In 27 cases altogether the value of $|\Delta_{max}|$ was 1 per cent or greater. It does not therefore seem possible to exclude any of the representations of the data as being outside the limits of observational error. The fact that the fit improves at most wave-lengths for higher values of τ_0 does not entitle us to prefer these larger values, since even for the small values a representation of the data within the observational uncertainty has been secured. The fitting also improves with the longer wave-lengths. On observational grounds it is probable that the measures at the shorter wave-lengths are more liable to error than those made at longer wave-lengths.

The variation of the parameters α', β', etc. with different assumed values of τ_0 is of the same general character at all wave-lengths. For very small values of τ_0, β' is negative (physically inadmissible), increases to zero, defining a minimum possible value of τ_0 and then reaches a roughly constant positive value. α' decreases slowly to a constant value, and γ' increases steadily with increasing τ_0. μ' decreases steadily to zero, giving a maximum admissible value of τ_0 and then enters the region of physically inadmissible solutions by becoming negative.

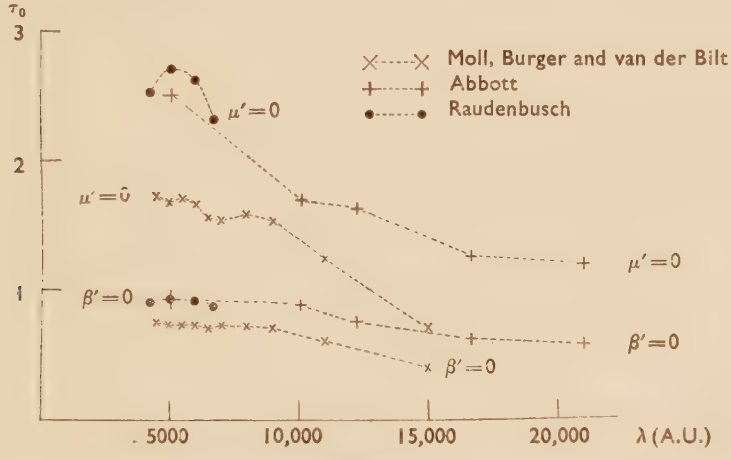

FIG. 3.

The general picture is illustrated in Fig. 3, which shows the limiting curves $\beta'=0$ and $\mu'=0$. It will be noted that while there is a considerable scatter for $\mu'=0$, there is a reasonable concordance for the curves $\beta'=0$. It will also be seen that in all cases both the maximum and the minimum possible values of τ_0 decrease for the longer wave-lengths.

Physically, in fact, since the top of the convection layer represents a definite geometrical level in the solar atmosphere, the variation with wave-length of the two limiting values means that, on some scale, the curve representing the variation of the opacity of solar material with wave-length must lie between the two limiting curves. This conclusion is a consequence of our general assumption about the form of the $B(\tau)$ curve, but, so far, does not lead us to any very definite information about the variation of solar opacity with wave-length. We shall presently conclude that the curves marked $\beta'=0$ are nearer the true state of

affairs in the solar atmosphere, and that these curves do therefore represent the variation of solar opacity through the spectrum. However, at the moment we have adduced no evidence which will enable us to reduce the ambiguity in the values of τ_0 and the dependent parameters α', β', μ' and γ'.

If we compare our results with those of Chalonge and Kourganoff we find that for the larger values of τ_0 our results represent merely an approximation to theirs but that there are large deviations if the smaller values of τ_0 are adopted.

5. *Application to Fraunhofer Lines.*—In an endeavour to remove the ambiguity from the results of our analysis of the continuous solar spectrum we now turn our attention to the behaviour of Fraunhofer lines across the solar disk. It may, at first sight, be objected that such a study involves so many disposable parameters that it cannot provide any very compelling evidence. However, there are certain cases in which it does seem likely that evidence about the behaviour of spectrum lines can provide a fairly definite lead in the discussion of the behaviour of the continuous spectrum. In particular, this is true of the observed phenomenon of the strengthening of weak lines towards the limb, first discovered by Miss Adam.* It is a statement probably not susceptible of strict proof, but one which will meet with general acceptance from those who have made experimental calculations in this field, that, on ordinary Fraunhofer line theory, no assumptions about line absorption coefficients alone can produce strengthening to the limb in any case in which $B(\tau)$ is strictly increasing with τ at all depths.† It seems improbable therefore that the distributions deduced by Chalonge and Kourganoff can produce the observed variations of line strengths, whereas our own analysis seems to hold out more hope of doing this. Our procedure will be to see how far our own analysis can reproduce the observed variations of line strengths, adopting the principle that where further assumptions may have to be made they must be justifiable as being physically consistent with the general picture which has already been sketched. It does seem that attempts which have been made so far to account for line strength variations‡ are open to the possibility of criticism on the grounds that they involve *ad hoc* mathematical assumptions, the physical bases for which are not clear.

We begin with the usual drastically simplified assumptions of current Fraunhofer line theory. Using the standard notation and taking $\eta =$ constant, and the form of $B(\tau)$ as given by equation (9), we find for the residual intensity in a direction at an angle θ to the normal to the solar surface the expression

$$r(\theta) = N(\theta)/D(\theta), \tag{17}$$

where

$$N(\theta) = \alpha' + \frac{\beta'}{\kappa^2}\cos\theta + e^{-\kappa^2\tau_0\sec\theta}\left(\mu' - \frac{\beta'}{\kappa^2}\cos\theta\right)\left(1 - \frac{\kappa^2-1}{\kappa^2 - 3\cos^2\theta}\right) + A_1\frac{\kappa^2-1}{\kappa^2-\sqrt{3}\kappa\cos\theta} + B_1\frac{\kappa^2-1}{\kappa^2+\sqrt{3}\kappa\cos\theta} \tag{18}$$

$$D(\theta) = \alpha' + \beta'\cos\theta + e^{\tau_0\sec\theta}(\mu' - \beta'\cos\theta) \tag{19}$$

and $\kappa^2 = 1 + \eta$, the notation being that of equations (11) and (12), and we have taken $\epsilon = 0$.

* M. G. Adam, Paper I, *loc. cit.*
† But see R. O. Redman, *M.N.*, **103**, 173, 1943.
‡ E. g. M. G. Adam, Paper II, *loc. cit.*

Further,

$$A_1 = \tfrac{1}{2}(\mu' - \beta'/q)e^{-q\tau_0}; \quad B_1 = \frac{1}{(1+2/q)}\{-(\alpha' - 2\beta'/q^2) - A_1(1-2/q)\}, \qquad (20)$$

where $q^2 = 3\kappa^2$.

The equations (20) are based on the boundary condition at $\tau = 0$ and on the continuity conditions at τ_0.

We shall sometimes use the notation

$$\rho = \beta'/\alpha' \; ; \quad \sigma = \mu'/\alpha'. \qquad (21)$$

These expressions have been studied extensively. Generally speaking, there is a tendency toward strengthening to the limb for the smaller values of τ_0 and weakening for the larger values. Small values of ρ and σ also tend to produce strengthening, whereas large values produce weakening. The tendency to strengthening is greater for the smaller values of η.

Physically, the reason for this state of affairs lies in the greater depth of formation of the weaker lines. To produce strengthening, the effective range from which contributions to $r(\theta)$ are drawn must include the top part of the convection zone. If a line is strong it will be formed wholly in the surface layer and will weaken steadily towards the limb. If the convection layer is at a small optical depth even fairly strong lines will be able to reach down to its level, and will show strengthening. If, on the other hand, τ_0 is large, none of the lines will be formed at all in the convection zone, and all will weaken towards the limb.

The observational evidence which is available consists of the observations of the variation of residual intensities across the disk in the $\lambda\lambda$ 3816–4227 region by Houtgast *, the eclipse observations of lines at the extreme limb by Redman for the λ 4200 region †, and Miss Adam's investigations of faint lines near the limb in the λ 4100 and λ 5100 regions.‡

Houtgast's observations give a general picture of the variation of residual intensities across the disk, whereas Redman's and Miss Adam's observations both deal with total intensities of faint lines at particular points on the solar disk, and are, therefore, from a number of points of view, less satisfactory for purposes of comparison than Houtgast's. In particular, the restriction to observations of total line intensity, inevitable for the weakest lines, presents a number of problems. We have no direct evidence at all as to the true contours of faint solar lines, since the observed contours are purely instrumental. We cannot, therefore, take a formula which purports to give residual intensity variations and combine its predictions into an expression for variation of total line intensity, for purposes of test. It is worth noting, however, that Houtgast's results show a strengthening ratio at $\cos\theta = 0.3$, which is as high as that observed by Miss Adam for total line intensities, only for residual intensities greater than about 90 per cent. This means that, on a purely observational basis, leaving aside the theoretical difficulties presently to be considered, if we accept both Houtgast's and Miss Adam's observations, then the true profiles of the lines which she observed must be shallow. This follows, since otherwise the strengthening of line depth observed by Houtgast for the higher residual intensities

* Dissertation, Utrecht, 1942 : see R. O. Redman, *loc. cit.*
† *Loc. cit.* ‡ *Loc. cit.*

would have to be compensated by the weakening or reduced strengthening of the line depths for the lower residual intensities when combining line depth data into data for total intensities of lines. It follows that in the true profiles of the faint lines there can be no small residual intensities, a result probably attributable to Doppler effect, either molecular or due to macroscopic movements of gases.

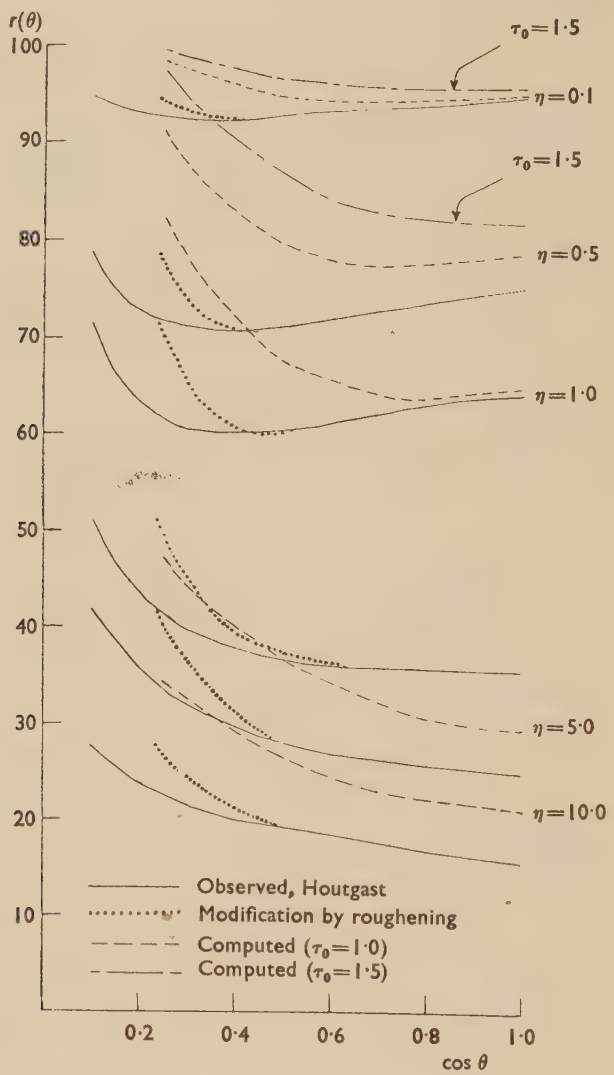

FIG. 4.

We now compare Houtgast's observations with the results deduced from the present theory. We take the observations of Raudenbusch at $\lambda 4260$ for which a solution is given by the values $\tau_0 = 1{\cdot}0$, $\rho = 1{\cdot}34$, $\sigma = 4{\cdot}38$. We compute formula (17) for the values $\eta = 0{\cdot}1$, $0{\cdot}5$, $1{\cdot}0$, $5{\cdot}0$ and $10{\cdot}0$. These are compared with Houtgast's observations in Fig. 4. At first sight the concordance looks very bad : the computed curves all turn upwards far too soon. However, there are important points of similarity : the lower computed curves all show a steady rise to the limb, while the upper ones show an initial fall followed by a rise. These

combined features are in fact exceedingly difficult to secure in any model. Experimental calculations show that the best fit is produced by taking values of τ_0 near the minimum possible; for example, taking the corresponding values $\tau_0 = 1{\cdot}5$, $\rho = 2{\cdot}25$, $\sigma = 2{\cdot}30$ (Fig. 4) merely worsens the fit by giving an increase of residual intensity to the limb for lines of all strengths.

Redman concluded that the effect of surface roughening might be approximately represented by replacing apparent cosine values by "true" values, according to the following scheme : as far as $\cos\theta = 0{\cdot}65$ the two sets are equal; thereafter there is a linear relation between apparent and true values, with a true cosine value of $0{\cdot}17$ at the apparent limb. A crude way to apply the correction for this effect is merely to transfer Houtgast's observed residual intensities to new cosine values according to the above scheme. This change produces some improvement in the concordance (Fig. 4). However, roughening, if present, will also affect the cosine values of observations of the continuous spectrum, and the correct way to take account of the effect will be as follows : first, the cosine changes must be applied to Raudenbusch's observations : these must then be analysed, and residual line intensities computed, after which a comparison with Houtgast's results, with modified cosine values, must be made. This corrects both sets of observations to the same basis. For Raudenbusch's observations at $\lambda 4260$, we obtain by this means the values $\tau_0 = 1{\cdot}00$, $\alpha' = 0$, $\beta' = 1{\cdot}10$, $\mu' = 0{\cdot}85$, and the computed residual intensities are shown in Fig. 5. The agreement with observation is fairly good, but not yet entirely satisfactory.

These parameters give a zero intensity at the true limb : as far as fitting the observations is concerned, a range of parameters ($\tau_0 = 0{\cdot}7$ to $1{\cdot}0$) is possible, giving an intensity at the true limb from zero to 20 per cent of the central solar intensity and not much variation in the computed residual intensity variations. Surface roughening applied to the continuous spectrum seems inevitably to imply a low intensity at the true limb, and this may be regarded as objectionable.* Such objections do not, however, seem insuperable. Even on a perfectly smooth Sun the observation of the actual surface layers, and the discrimination between a discontinuous fall and a smooth steep fall of intensity there, would be very difficult. Further, if roughening modifies the cosine values in the manner suggested by Redman, only a most unexpected form of extrapolation to the true limb can keep the intensity there high. Finally, if even no more than a slight degree of roughening is present, the true limb will be unobservable and, in consequence, no direct observational evidence about solar surface conditions will be obtainable.

It must be noted that Redman's analysis presumes a wavy solar surface, with optically thick waves, but if this picture has to be replaced by another, e. g. by a spiky surface resembling grass on a lawn, the simple relation between true and apparent cosine values used here may have to be replaced by something much more complicated.

However, at the moment it seems that we must be content with Redman's formulation and must study other methods of improving the accordance between theory and observation. So far we have taken η constant. For consistency with our present picture it seems that we cannot permit ourselves to consider an arbitrary variation of η but must take η constant in the convection zone where mechanical mixing tends to produce an averaging of physical conditions with

* See R. O. Redman, *loc. cit.*

depth. Models having constant but unequal values of η in the two layers have been tried, but it turns out that these cannot produce the slower increase in residual intensity near the extreme limb which observation requires. Near the limb, line formation takes place wholly in the surface layer, and the character of residual intensity variation is produced almost entirely by the value of η in the surface layer. An alternative idea is the superposition of a thin scattering atmosphere,

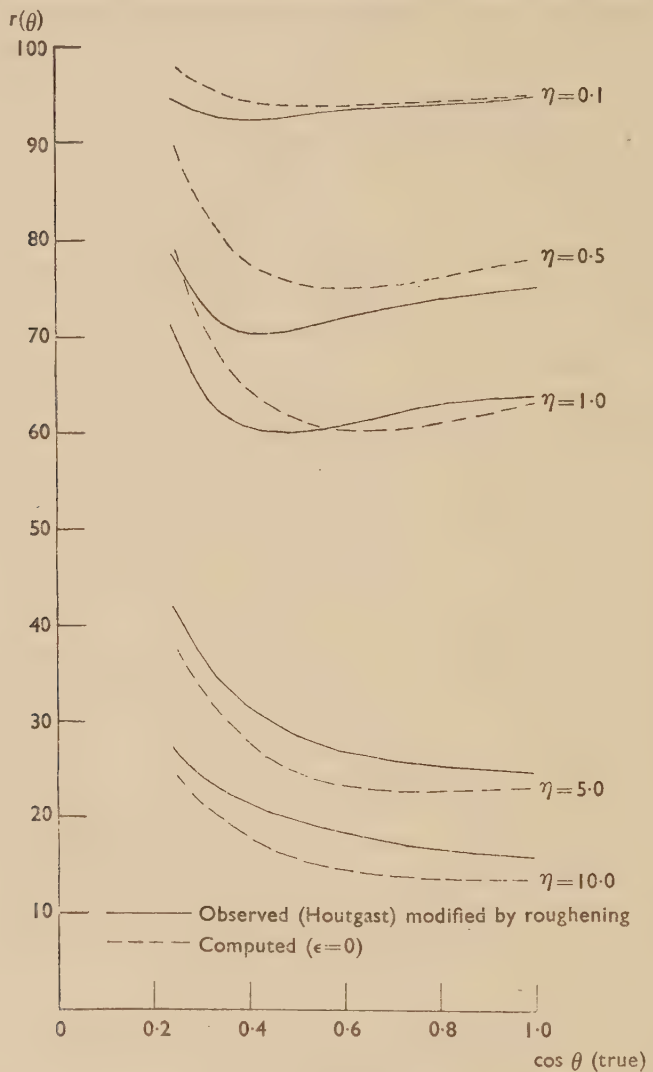

Fig. 5.

scattering either generally or only in the line frequencies, and this can effect some improvement of fit, but is not completely satisfactory. In particular, if we assume a selective scattering by a layer of atoms above the photosphere which are chemically identical with those producing the absorption line, we must assume that the scattering coefficient in this layer changes with wave-length variation in much the same way as η changes for the ordinary absorption line. We can improve the fit for the variation of residual intensity in the wing of a line by assuming a small selective scattering coefficient, but this would seem to

imply the presence of a large selective scattering coefficient near the line centre, which will ruin the agreement for strong lines (or small residual intensities) already obtained.

The only possibility seems to be to assume the existence of a small uniform scattering both within the line and in the neighbouring continuous spectrum. As an example, an optical depth in the scattering atmosphere of 0·1 improves the fit for weak lines, without spoiling it for the strong ones, and without spoiling the continuous spectrum fit. However, even this is not a completely satisfactory solution. For $\eta = 0{\cdot}1$ the concordance is improved but the line depth still begins to increase at too large a value of $\cos\theta$.

The only remaining obvious possibility is to consider the effect of changing ϵ (as defined by Eddington), which so far has been taken as zero at all depths. We shall keep ϵ constant with depth in what follows. Now, in general, if we take $\epsilon = 1$, all spectrum lines will vanish at the limb, and in many models any increase of ϵ has the effect of making line weakening to the limb more rapid. This is not the case, however, if the intensity in the continuous spectrum at the limb is small or zero. If we compute for $\tau_0 = 1{\cdot}0$, $\alpha' = 0$, $\beta' = 1{\cdot}10$, $\mu' = 0{\cdot}85$ (the parameters derived from Raudenbusch's observations of the continuum at $\lambda 4260$ as modified by roughening), taking $\epsilon = 0{\cdot}5$, we obtain the curves shown in Fig. 6. All the higher residual intensity curves are in excellent agreement with the observations : the topmost curve ($\eta = 0{\cdot}1$) gives strengthening in agreement with the strengthening observed by Miss Adam for total line intensities at $\lambda 4100$ and at the correct (true) cosine value. Redman's observation that the lines at an apparent cosine value of 0·05 (true value 0·21) are as strong, or even slightly stronger, than at the centre of the disk, is also reproduced on this model. The lower residual intensities give curves in agreement with observation at the limb, but which are too flat near the centre of the disk. The fit for strong lines for $\epsilon = 0$, as shown in Fig. 5, is in fact better, and within the observational errors. It is not, of course, necessary that ϵ should have the same value for lines of all strengths. It does not seem worth while to go further in attempting to improve the observational fit.

We effect a comparison with Miss Adam's observations at $\lambda 5100$ by analysing Raudenbusch's data for $\lambda 5060$ as corrected by roughening. The results for $\alpha' = 0{\cdot}1178$, $\beta' = 0{\cdot}9281$, $\mu' = 0{\cdot}8033$, $\tau_0 = 1{\cdot}0$, $\eta = 0{\cdot}1$ with $\epsilon = 1$ are shown in Fig. 7. It will be seen that strengthening in a ratio of 1·5 : 1 occurs, in agreement with Miss Adam's observations for total line intensities of faint lines at this wave-length.

It is convenient at this point to summarize the work and to attempt to assess it critically. It is nothing particularly novel that models can be devised which fit the observations of Fraunhofer line variation across the disk, but if we consider these models we find that they involve special hypotheses about the variation of η which it seems difficult to reconcile with general theoretical notions. Further, any theory which does depend on artificial distributions of η of this kind, in reality ascribes strengthening of faint lines to the limb to the specialized cause (whatever it may be), which does produce these rather unexpected distributions of line absorption in optical depth. But, as far as it is possible to judge, line strengthening in the visible region is a general phenomenon affecting a large proportion of the faint lines in a similar way. Our approximation $\eta =$ constant must be regarded, not so much as a mathematical hypothesis, as an attempt

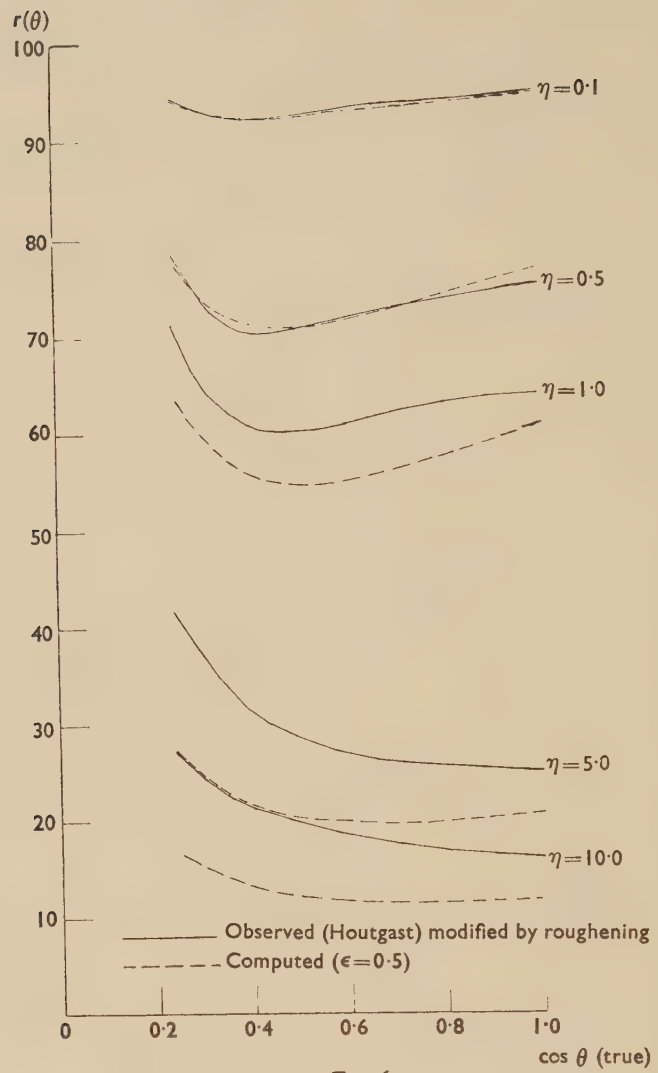

Fig. 6.

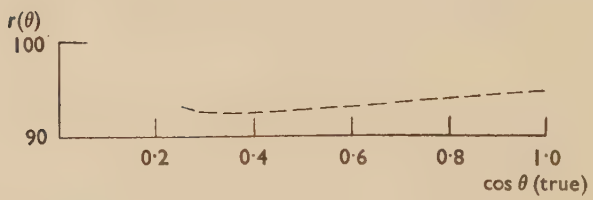

Fig. 7.

to account for line behaviour solely as an expression of average depth of line formation. It seems possible that minor differences such as, for example, the possible differential behaviour between abundant element lines and rare element lines found by Miss Adam*, might be accountable as due to variations in ϵ, a hypothesis which seems logically and physically self-consistent.

* Paper II, *loc. cit.*

What we have done is to make a hypothesis about the character of temperature variation with depth in the solar atmosphere, to attempt to justify it on physical grounds, and to follow out its consequences. By considering the continuum alone we find that certain limitations must be placed on the parameters involved in the picture to produce agreement with observation. Then, turning to absorption lines, we find ourselves compelled to introduce Redman's roughening effect, after which, within the limits imposed by our general hypothesis, we find it possible to account for Fraunhofer line variation solely as a consequence of differences in effective depth of formation, plus variations in ϵ. If we consider the rôle which roughening plays in this analysis we find that it operates in a number of ways. First, it seems practically to compel us to adopt low values for the intensity at the true limb, and secondly, it prevents us from ever observing the true limb. Thirdly, as a consequence of these low limb values it permits increases in ϵ to give the necessary degree of line strengthening to the limb.

The picture is at least self-consistent, and it might be maintained that it has a certain merit, in that it achieves its ends by the use of a few general hypotheses (adoption of a certain general type of $B(\tau)$ curve plus surface roughening), rather than by specialized *ad hoc* assumptions. It is nevertheless an unorthodox picture, presenting as it does an atmosphere composed of two layers, in one of which there is an averaging of conditions by ordered convection, while in the second surface layer, the temperature gradient is fairly small, as a result, perhaps, of turbulent mixing of gases. The stratification is produced not statically but by motions somewhat analogous to the ordered motion of water over a weir, producing turbulent mixing in the river below.

There is, however, no guarantee that the picture, self-consistent though it may be, is a true description of the solar atmosphere, but the function of any theory ought to include the suggestion of methods for its observational test, and, in the present case, it does seem that a fairly definite test is possible. As we have seen, we must, in Fig. 3, prefer the lower possible values of τ_0 at each wave-length, and these decrease as the wave-length increases. The data of Fig. 4 will require some modification if roughening is included, leading to a slight reduction in the minimum values of τ_0 and a large reduction in the maximum values. We may therefore take it that the curves marked $\beta' = 0$ are not far from the curve of variation of τ_0 which we ought to adopt, and, therefore, in the infra-red region we are led to expect greatly reduced values of τ_0. As we have seen, formulae (17)–(20) are such that reduction of τ_0 tends to accentuate line strengthening to the limb. Our picture therefore suggests that in the infra-red line strengthening should be much more common than in the visible region.

In an attempt to give a more precise form to this expectation, we first compute residual intensity variations for λ11,000 and λ20,970, neglecting roughening, using minimum values for τ_0 and taking $\epsilon = 0$. The results for λ11,000 are shown in Fig. 8, and it will be seen that strength changes to the limb are small. At the longer wave-length there is a general strengthening to the limb. For the same values of η the residual intensities computed for these wave-lengths are higher, especially for $\eta = 5$ and $\eta = 10$, than the corresponding ones in the visible region. This is in accordance with the observed fact that very strong lines are rare in the infra-red.

In the case of λ11,000 we have attempted to go further by introducing Redman's correction for roughening. The results are shown in Fig. 9. The

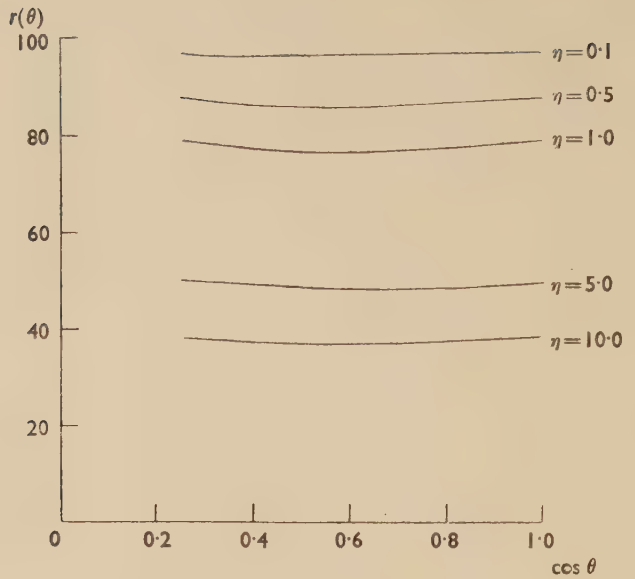

FIG. 8.—*Expected line variation in infra-red.*
(λ=11,000)

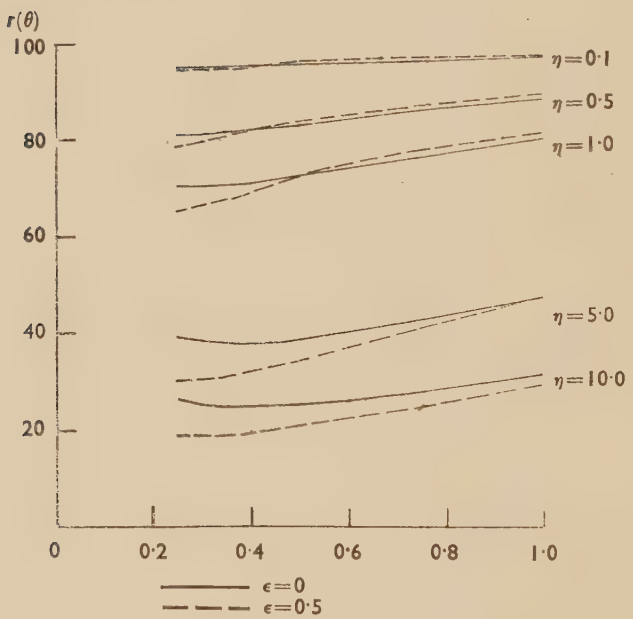

FIG. 9.—*Expected line variation including roughening.*
(λ=11,000)

general expectation is, therefore, that in the infra-red all lines will maintain at least their central intensity and may even show strengthening to the limb. Lines of hydrogen will probably be anomalous in behaviour, and may have to be excepted from this general expectation.

If the expected behaviour is actually observed, it may then be possible to re-discuss the present topic with rather more precision, since at least some of the many uncertainties by which it is beset may be eliminated.

I am particularly indebted to Dr Redman for the opportunity of discussing these problems at length with him.

Radcliffe Observatory,
Pretoria,
S. Africa :
1947 *June* 12.

SOLAR HYDROGEN CONVECTION AND ORIGINS OF THE ULTRA-VIOLET STIMULUS FOR COMETARY AND TERRESTRIAL SPECTRA AND THE LOWER IONOSPHERE

Martin Johnson

(Received 1947 October 17)

Summary

Eddington's theory, that Lyman radiation is liberated at the top of Unsöld's convection zone and can reach the photosphere in excess of the Planck distribution, is investigated by calculating the continuum between 600 A. and 900 A. in an electron capture spectrum by Cillié's methods. Some conditions are specified for deciding whether such radiation can escape from the Sun and supply the excess ultra-violet demanded by cometary spectra in CO^+ and N_2^+, and the several likely excitations in the D region of the ionosphere accompanying solar flares. In particular the optical depth of the electron captures decides the extent to which the radiation becomes redistributed into a Planck spectrum, and this will fluctuate during certain solar disturbances; the resulting changes in extreme ultra-violet distribution are large compared with change in total energy output.

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1. *Introduction.*—A variety of workers have presented facts considered as implying that radiation between 600 A. and 900 A. escapes from the Sun with intensity many times that allowed to such high frequencies in a Planck distribution at 6000 deg.; an early example is Saha's (1937) (**1**) correlation of ionospherics and spectra, and a recent example is Kiepenheuer's (1947) (**2**) coronal theory of ionospherics, while notable between these is Swings' (**3**) report commenting on the high excitations in cometary spectra. Hunter (1943) (**4**) systematized the grounds on which such excess solar ultra-violet is demanded. Suggested means of fulfilling these demands have included "windows" emitting subphotospheric radiation at 800 A. or even 10 A. or 2 A. (Maris and Hulburt, Vegard, Preston), and also degradation products of Edlén's corona (Waldmeier).

It is by no means certain which, if any, of the demands are inescapable: some writers claim a 10^6-fold excess, whereas Woolley (**5**) has formulated conditions which would have to be fulfilled if the upper ionosphere were due to O_1^+ excited without great divergence from Planck's ultra-violet. But the uncertainty emphasizes the need for exploring all possible solar mechanisms of super-Planck distribution in the ultra-violet during transient departure from equilibrium, hitherto commonly left in the stage of qualitative suggestion. One possibility, originating independently of the above needs and also independently of the coronal excitation at far higher voltages, is implicit in Redman's (**6**) linking of high frequency solar emissions with the excess ultra-violet which Eddington associated with Unsöld's convection zone. We here investigate this implication, since, in view of Plaskett's and Thackeray's elevating of solar convection nearly to $\tau = 0$, any energy liberated there in excess of normal photospheric may escape and contribute to terrestrial and other extra-solar excitation. In particular, if

Eddington's ultra-violet were to provide for steady cometary and night-sky terrestrial phenomena, fluctuations in the τ level at the top of the Unsöld region may be found relevant to understanding the brief transients of the Dellinger and Newton flares, with their important consequences in the D phenomena of the ionosphere and accompanying radio fading.

2. *Extra-solar consequences of excess ultra-violet radiation.*—We begin by extending any list, such as compiled by Hunter, into a classification of grounds for extracting an ultra-violet band common to the several sources of evidence. In the Sun itself, normal energy levels in the reversing layer extend up to about 5 electron-volts, in accord with a Planck photosphere at 6000 deg.; significant *local* exceptions are *He* absorptions implying 21 e.V. (548A.) above disturbed spot regions, and He^+ in chromosphere and prominences implying 25 and more electron-volts. The quite different orders of magnitude of Edlén's coronal lines are not within the scope of the present discussion, whether their origin be nuclear or from lateral discharges.

(i) Extra-solar spectroscopic evidence.

(*a*) Comets (**7**): The probable situation is of meteoric particles, liberating at perihelion occluded gases polyatomic in *H*, *C*, *O*, *N*, which yield e. g. *CH* near the nucleus and C_2 and *CN* over the head; excitations of these by resonance-fluorescence mechanisms supply the observed bands. Diatomics photoionized include CH^+ (requiring 11·1 e.V. or 1110 A.) in the near tails, and along the tails CO^+ (14·1 e.V., 880 A.) and N_2^+ (15·5 to 18·7, 798 A. to 660 A. according to excitation). Herzberg's CH_2, also *OH*, *NH*, NH_2, and possible parent molecules CO_2, NH_3, H_2O, CH_4, present no higher excitations; with the detailed band analysis elucidating extremes of low pressure and temperature (Swings, McKellar, Bobrovnikoff, Herzberg), they contribute to the problem of excess ultra-violet only the fact that radiation for final products CO^+ and N_2^+ is unobtainable in any dominant intensity from a normal Planck spectrum at mean solar temperature.

(*b*) Twilight flash: This transient dawn and dusk strengthening of λ3914 to dominance of the terrestrial night-sky or "non-polar aurora", first discovered by Slipher (**8**), is important as indicating excited N_2^+ lost and revived at extinction and reappearance of sunlight. It is therefore not subject to the doubtful possibility (e. g. Wurm, 1939) of a corpuscular explanation of the cometary bands. The convincing spectrograms (e.g. Elvey, 1942 (**9**)) distinguish this transient photo-effect from the permanent non-polar night-sky (λ5577 and 6300 of O_1 stronger) and the polar aurora whose excitation is more probably corpuscular (λ5577 and 3914 both strong).

(ii) Ionospheric evidence.

Here the energy levels are by implication only, since the night-sky spectra appear in general above the ionosphere when they can be localized in height. The most comprehensive treatment of the molecular problems is by Bates and Massey (1946) (**10**), giving not only their own ascriptions to O_2^+, O_1^+, N_2^+, of the several regions of maximum intensity, but also the different ascriptions of other authors.

(*a*) Daily and seasonally varying upper regions including the very irregular F_2: these are mainly photoionized but with some possible corpuscular contribution at the top, in the environment where $O_1/O_2 \gg 1$ as first suggested by Chapman and developed by Wulf and Deming (**11**). Woolley (1947) (**12**) exhibits this atomic character graphically as a rise in oxygen dissociation from $\frac{1}{10}$ to 10 between 90 and 110 km.

(*b*) E, with variations often resembling those of the less unsteady F_1 but including the sporadic E which may involve meteoric excitation: this shallow region is associated with the transition where O_1/O_2 is approximately unity.

(*c*) D region of briefly transient phenomena: this exhibits a weak background intensified violently during solar flares, and many of its features are followed by delayed geomagnetic effects (Newton) (**13**), indicating primary photoionization in the lowest zone where $O_1/O_2 \ll 1$, e. g. 70 km. Classical collections of data are by Budden, Ratcliffe and collaborators (**14**), using the brief lowering of long-wave reflections, by Berkner and collaborators (**15**) using quite different methods, and by the Australian (**16**) observers with again a different technique. The peculiar difficulties facing all attempts to understand penetration of F and E by rays capable of ionizing D are referred to later.

Omitting low-voltage excitations, the ionization potentials relevant to all these phenomena are:

CH (11·1),
CO (14·1),
O_1 (minimum 13·5, excited 16·9, 18·5),
O_2 (minimum 12·2, excited 16·1, 16·9, 18·2),
N_2 (minimum 15·5, excited 18·7).

N is more readily ionized than dissociated, so that upper ionospheric O_1 is not accompanied by similar N_1, but both diatomics follow experimentally the tendency not to absorb at the minimum frequency and to ionize into excited states for the second electron.

These data collectively demand a solar source rich between 600 A. and 900 A. but not accompanied by general ultra-violet, since, e. g. that would cause more double ionization of metals than is seen in the reversing layer. In view of the many estimates of actual intensity required, often stated as 10^3–10^6 times that of the normal photospheric component, we shall investigate reasons for the Sun to emit up to 10^5 times Planck's intensity in this band, with fluctuations not solely due to temperature change.

3. *Scope and limitation of Eddington's source of ultra-violet.*—(i) As initial step towards examining Eddington's theory of recombination at the top of the convection layer as possible source of the required intensities, we first extract the physical grounds for his view that radiation is there liberated in excess of that demanded by thermal equilibrium and ionization equilibrium. In addition to Eddington's papers (**17**) the exposition by Hunter (**18**) may be consulted. The relevant steps may be summarized as follows.

(*a*) Convection arises between, say, 9000 deg. and 15,000 deg., because the employment of energy in altering the degree of ionization from so small to so large a fraction at 13·5 e.V. simulates an enhancing of specific heat, depressing the effective ratio γ by Fowler and Guggenheim's process. (*b*) For even slow resulting convective drift the degree of ionization lags behind change in T and p. (*c*) Opacity to $\lambda > 910$ A. is thereby reduced for both ascending and descending currents, while to $\lambda < 910$ A. it is large for descending currents but small for ascending; the net effect increases the transmission of high frequency radiation from the hotter base of the layer. High frequencies are also generated as a capture spectrum at $\lambda < 910$, at the summit of the convection column, by the arrival there of the superabundant H_1^+ available for recombination in excess of that which would have been in equilibrium at the lower temperature. It is these excesses which

Hunter and Redman quoted in connection with the ultra-violet needs of the extra-solar phenomena, but which seem not to have been pursued quantitatively, or to have been regarded as involving local and temporal fluctuations.

(ii) This omission is probably because the amount of the excess has remained uncertain; it must depend upon correct separation of the attainment of equilibria in ionization, temperature, and mass-motion, and will be complicated by the H^- ion production whose importance has been recognized since Eddington wrote. The actual lag in subphotospheric recombination of convected hydrogen was never brought by Eddington to quantitative estimation of the super-Planck distribution of ultra-violet, and present policy must be to enquire what ranging of variables would allow this distribution to be of practical interest. Schwarzschild (**19**), by adding collision ionizations, limits some of the first conclusions to the very top of the Unsöld region; but it is just there that the excess capture spectrum arises. Indeed, Eddington finally considered that Schwarzschild had localized the excess of the highest frequencies in a shallow layer, intensifying the extreme ultra-violet only. It is probable that the remainder of the theory, giving "window" transmission of a general thermal radiation from below, may be found to provide less than his first paper suggested.

4. *Spectral redistribution.*—In addition to this uncertainty in quantitative estimates of the intensities generated *in situ*, the problem of spectral redistribution before the radiation can leave the Sun has not been attempted and must depend on principles such as those of Greaves' (**20**) "escape of continuous radiation from stars". Some progress can be initiated by distinguishing two limiting cases as follows, and asking what conditions would have to be fulfilled for the radiation to be generated and then to escape in sufficient quantity.

Case (i). If Eddington's excess of H_1^+ were neutralized in a very thin photospheric layer, above which $\rho \to 0$ in the general expression for optical depth

$$\tau = \int k\rho \, dx,$$

then to a close approximation

$$I_\nu(\text{emergent}) = I_\nu(\text{capture}).$$

The distribution of intensities along the capture spectrum at any particular series limit is given by Cillié in papers (**21**) based on the probabilities of encounter between ion and electron worked out by Gaunt and others,

$$E_\nu \, d\nu = n_+ n_- \left[\frac{2^6 \pi^5}{(6\pi)^{\frac{3}{2}}} \frac{\epsilon^{10}}{m^2 c^3 h^2} \right] \left(\frac{m}{kT} \right)^{\frac{3}{2}} e^{-\frac{h(\nu - \nu_0)}{kT}} \, d\nu.$$

In this expression, m, ϵ, c, h, k, have their usual meanings in atomic physics, and ν_0 is the series limit, here the Lyman limit at 13·5 e.V. (910 A.). n_+ and n_- are concentrations per cm.^{3} of protons and electrons leading on recombination to a liberation of energy density $E_\nu \, d\nu$ which we can transform by $(c/4\pi)$ into an intensity in ergs per cm.^{2} per sec. In our case

$$I_\nu \, d\nu(\text{capture}) = f_1 \frac{c}{4\pi} E_\nu \, d\nu,$$

the fraction f_1 representing the proportion of captures into the lowest energy level, generating the continuum of highest frequencies. Further cases might be taken by adaptation to the discrete Lyman lines 1215 A. etc. or to Balmer frequencies or to the helium capture limit at 506 A. or helium lines 584 A. etc.; but since

requirements for the present phenomena belong mainly between 600 A. and 900 A. we use only the Lyman continuum, retaining the fraction f_1 as reminder that only upper limits to the energies are obtained by ignoring captures into subsidiary levels.

Case (ii). Although we shall suggest that some kinds of solar transients mean occasional brief approximation to Case (i), in general Eddington's recombination spectrum will be liberated at $\tau > 0$: before emergence of its energy from the Sun there will have been a degradation through repeated absorptions and re-emissions according to ρ and to each k_ν. If τ is sufficiently deep, the emergent radiation will have little of the initial distribution and will approximate to the universal Planck type; in the limit it will become a thermal spectrum at enhanced temperature T', and the previous equation is replaced by

$$I_\nu(\text{emergent}) = I_\nu(\text{Planck})_{[T']}$$

for all values of ν. It is convenient to define a second fraction f_2; this is strictly a degradation function such that

$$f_2 = 1 \text{ at } \tau = 0,$$
$$f_2 = 0 \text{ at } \tau = \tau',$$

where τ' is the optical depth at which absorption coefficients Σk_ν forbid survival of the Cillié distribution. In the limit, the radiation leaving the Sun is given by

$$I_\nu d\nu = (2h\nu^3/c^2)d\nu \Big/ \left(e^{\frac{h\nu}{kT'}} - 1\right).$$

But when τ is small, it will be sufficient to ignore the functional form of f_2, and to treat it as a fractional multiplier, so that

$$I_\nu d\nu = f_1 f_2 \frac{c}{4\pi} E_\nu d\nu.$$

We consider in Section 7, below, evidence for irregular oscillation of the solar facts between the extremes represented by these Cases (i) and (ii), but meanwhile the energy output of each must be calculated.

5. *Graphical comparison of capture spectrum with thermal spectrum.*—(i) Cillié's equation, above, is obtained by balancing ionizations and captures of Maxwellian electrons, but has been applied successfully by workers (**22**) in non-equilibrium phenomena. We compute from it the plots of $\log_{10} I_\nu d\nu$ in ergs per cm.^{2} per sec. between 600 A. and 900 A., shown in Fig. 1, for $d\nu = \nu/100$. The separate lines are for values of the principal unknown, $n_+ n_-$, of 10^{20}, 10^{22}, 10^{24}, 10^{26}. For comparison, e. g. with "window" theories quoted above, and with the empirical demands for radiation far more intense than provided by Planck components at solar temperatures, we plot thermal distributions in the same units and the same frequencies at 6000 deg., 9000 deg., 12,000 deg. and 20,000 deg. The capture spectrum, less temperature-sensitive, is plotted for a photospheric 6000 deg. and for the 9000 deg. earlier suggested as summit of Unsöld's original layer. Since the terrestrial phenomena have been held by many to demand intensities of such order, we impose a heavy line on the plots at 10^5 times the photospheric Planck intensity.

(ii) The diagram indicates a critical density when $n_+ n_-$ slightly exceeds 10^{22}. At 10^{22} itself for 6000 deg. the capture radiation is between 10^4 and 10^5 that of Planck's 6000 deg; at 9000 deg. the capture radiation at 10^{22} exceeds 10^5 times Planck's 6000 deg. for $\lambda < 800$ A. and falls below for longer λ. This places the

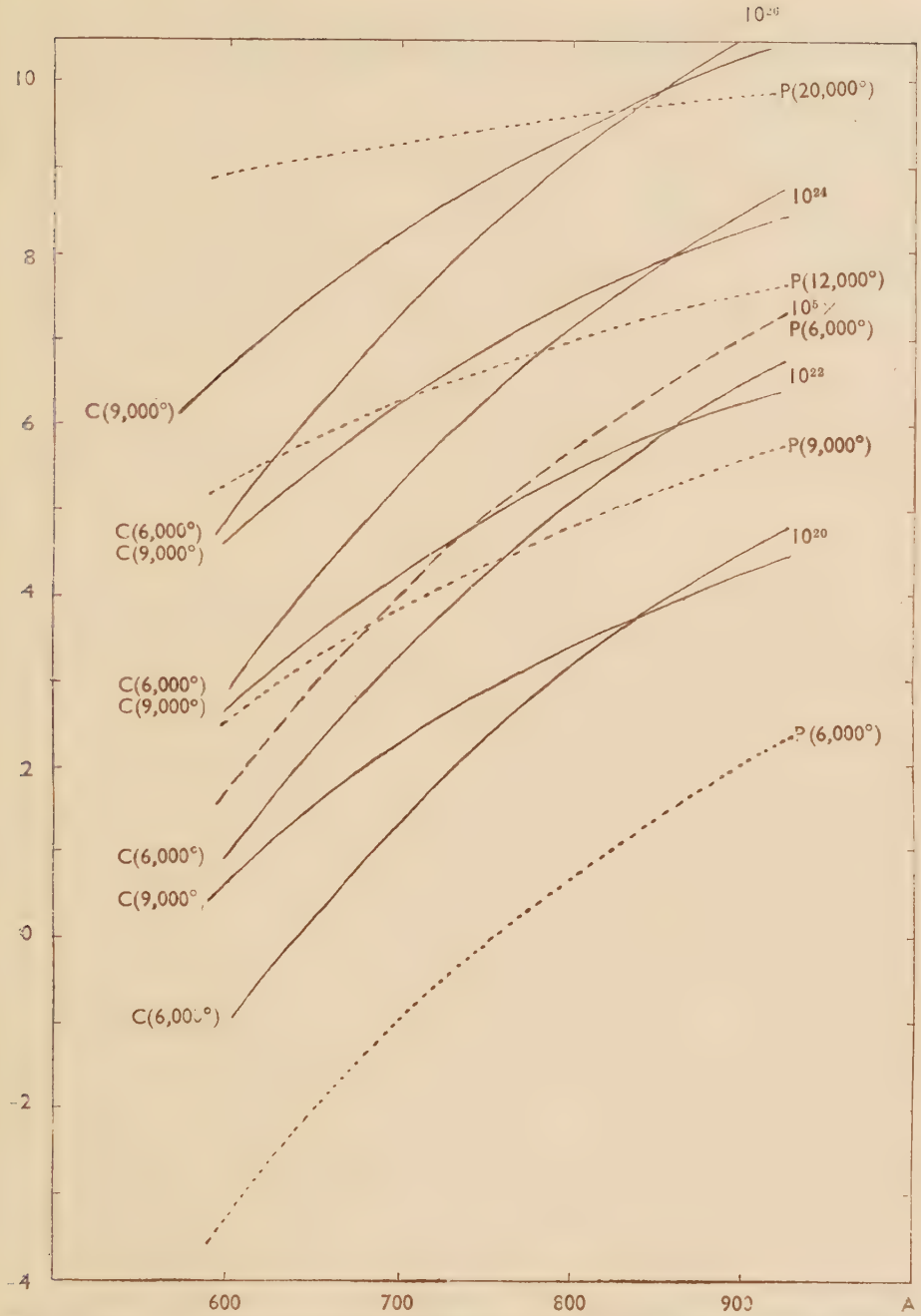

FIG. 1.—*Log* $_{10} I_\nu \, d\nu$ *ergs per cm.*^{2} *per sec. in Planck spectrum* (*P*) *and in capture spectrum* (*C*).

emission from n_+n_- around 10^{22} and $f_1f_2 \to 1$ rather above that of Planck's at 9000 deg., showing what "window" theories would require if their excess in lower frequencies were not an obstacle to their acceptance.

6. *Hydrogen density and electron density.*—Such a diagram, showing that as $f_1f_2 \to 1$ the capture spectrum could fulfil the demands of 10^5-fold excess ultra-violet when $n_+n_- \to 10^{22}$, now introduces the question of conditions under which

concentrations of this order might be expected. Wildt's (1947) (**23**) analysis of visible Balmer radiation from the chromosphere assigns to H_1 an abundance there of more than 10^{15} per cm.^{3}, of which a fraction 10^{-4} is ionized, providing a likely n_+ as small multiple of 10^{11}; if n_- were solely drawn from the electrons of this hydrogen it would also be 10^{11}, but if drawn from the more easily ionized accompanying metals whose abundance might be 10^{-4} that of hydrogen and all ionized to first stage only, the same result of $n_+n_- = 10^{22}$ would be reached. In the photosphere and subphotosphere, densities are far greater but hydrogen abundance is probably not increased as much as total density. Eddington estimated n_+ for the summit of a convection column as $>10^{16}$, but this would be considerably below the photosphere: τ would not there approach zero nor f_2 unity, except when hydrodynamic surges lifted the convected mass momentarily to near surface. But that is just the occurrence which we suggest as possibly underlying some solar flares. For utilizing our graphs it may be noted that radiation comparable in spectral distribution with Cillié's would emerge if the variables altered approximately at the rate

$$\frac{n_+}{n_+\,(\text{chrom.})} = \frac{1}{f_2}.$$

7. *Depth of origin and solar values of k.*—(i) Two sets of facts offer information as to the depth of origin controlling the spectral form in which Eddington's radiation might emerge. Consider the eruptive states of the Sun. (*a*) Since solar flares are notoriously unaccompanied by any great super-photospheric temperatures surrounding them, τ on those occasions is too shallow for degradation towards Planck redistribution (Case (ii)) and lateral spread to have occurred; the extra-solar phenomena depending upon the band between 500 A. and 1000 A. suggest escape in a form not very far from the approximation of Case (i), with f_2 nearly unity and even f_1 not small. (*b*) A non-zero limit is, however, set to diminishing τ, since a variety of low excitation derivatives appear in Allen's (**24**) complex flare spectra, as well as the commonly observed bright $H\alpha$ and Ca^+. Newton's (**25**) geomagnetic effects after 24 hour lag, indicating corpuscular activity, also the expenditure of some flare energy upon 400 km./sec. rise and fall of material, and the Brück-Moss motions of chromospheric $H\alpha$, are among the proofs that energy is scattered by absorptions which cannot leave a pure capture spectrum to emerge even when the situation most closely approaches Case (i).

(ii) Actual estimates of the normal summit of a convection column have been in terms of an optical τ, whose associated k is that accounted for by the negative ion absorptions of Massey and Bates (**26**), Chandrasekhar, Wildt and others, covering $\lambda > 4000$ A. In the near ultra-violet, metallic series limits may replace H^- as absorbing mechanism, but Greaves (**27**) has emphasized that only a fortunate accident enabled all these to be included under an apparently constant mean k. This constancy will not hold far outside, especially in the Lyman region, the appropriate k being nearer to that which must control the atmospheric gradients of an A-type or B-type star. The Sun cannot suddenly adjust itself to local equilibrium with a radiation field over-rich in 600 A. to 900 A. Observed chromospheric motions are part of the attempt at such adjustment, but meanwhile Lyman radiation in excess of normal may be reaching the exterior with far greater or far lesser facility than via the gradients of any star's steady state. Estimates of k and τ in the visual region can therefore provide only the roughest of hints towards the optical depths relevant to the present problem. Such estimates of τ range from Unsöld's (**28**) original $\tau = 3$ for a summit of convective regions, to Plaskett's (**29**) $\tau \doteqdot 1$ in

discussing observed granulation, and Thackeray's (**30**) $\tau = \frac{2}{3}$ in a disproof of the usual deduction of radiative equilibrium by observed limb darkening. In the last case, even visual radiation from a convective region could emerge with $I = \frac{1}{2}I_0$, since $e^{-0.67} = 0.51$.

(iii) These considerations make possible a picture of rapid temporary fluctuations from any normal value of τ, or optical depth at which the capture spectrum originates, during even slight solar disturbance. In effect the conditions can switch from near Case (ii) to near Case (i) during surges no more violent than must underlie some spots or invisible centres of excitation. Whenever locally τ approaches zero, the excess Planck distribution reverts towards the primitive Cillié distribution by rise in the value of f_2. Such a rise to $\frac{1}{10}$ or $\frac{1}{2}$ with f_1 not far from unity would allow a hydrogen concentration reaching between twice and ten times that of the chromosphere to provide extra-solar transmission of Lyman radiation to the 10^5-fold excess intensity sometimes demanded. Under quite a small fraction of the solar surface a surge locally increasing the proton or electron concentration and momentarily diminishing the value of τ will increase by several times the Cillié components at the expense of Planck's in the radiation terrestrially intercepted. These seem not impossible achievements even without reaching Eddington's value of $n_+ > 10^{16}$; they may underlie the causation of some brief flares which affect profoundly the ionosphere.

8. *Extra-solar effects.*—In distinguishing between extra-solar facts attributable to semi-permanent convective regions whose capture spectra degenerate into nearly Planck radiation, and those which are brief and exhibit the rare emergence of an ultra-violet more closely resembling Cillié's spectra, we encounter as follows the special question of how radiations can penetrate to the lower or D level in the ionosphere. The foregoing outline of suggested conditions which would permit both generation and escape of a 10^5-fold excess in the spectral region 600–900 A. may have direct application to the quoted cometary excitation of CO^+ at 14·1 e.V. and N_2^+ at 18·7. No opacity of external intervening material * introduces any insuperable difficulty as to loss of initial intensities beyond inverse square attenuation. But in any suggestion that this capture spectrum might supply the "missing rays" for exciting the ionospheric D region, we have to face the incompatibilities common to all theories of how the 70 km. level can respond to transient solar flares; it appears to need a spectrum whose relevant frequencies ought not to have survived transmission through the E and F layers extending over several maxima to 200–500 km. above.

Two difficulties particularly have been most clearly exposed by Berkner and by Massey respectively, and call for scrutiny before any attempt at introducing capture radiation as discriminant between alternatives.

* *Note added* 1948 *January* 3.—I am indebted to the discussion of the paper for Fellows' emphasis that much of the consequences will depend on the *solar* levels at which opacity to Lyman radiation sets in. In particular, Professor Cowling drew attention to the consideration that for some ultraviolet frequencies "photosphere" may denote a region well up in the chromosphere. If the photosphere is defined by its opacities, the layer contemplated by Eddington may occur higher relative to some regions of line absorption than was considered when his papers were written. The main argument of the present paper could be applicable to any such level, indeed to any region not so deeply buried as to allow solely black distribution to emerge. At certain heights, where there is sharp transition from general to selective opacity, the Eddington emissions would be supplemented by re-radiations of the kind usually considered chromospheric, but after stimulus in this case from immediately underlying Lyman continua; this would insert intermediate stages of fluorescence or resonance between the source which Eddington postulated and the final escape from the Sun's vicinity.

(*a*) Massey (**31**) has emphasized that if heights are correctly estimated from scale-height, and if densities and theoretical cross-sections are reliable, no component capable of absoption in O_1, even up to 20 e.V., can penetrate below the atomic zone; so that D (and even normal E) regions are only attributable (apart from rare gases) to O_2 ionized by the narrow band between 12·2 and 13·5 e.V., unless theories of pre-ionization (Nicolet) prove to be a solution.

But in considering alternatives, it must be noted that experimental work on common diatomics yields slight absorption by the molecule at minimum potential, O_2^+ appearing at 16·1 and 18·2 rather than at 12·2, and N_2^+ at 18·7 rather than 15·5. This is a generalization stressed by Saha (**32**), and by Chapman and Price (**33**), and recognized in Bates' and Massey's cross-sections.

(*b*) The common statement that E and F ionizations remain unaltered during the instant response of underlying D to chromospheric eruption, can misinterpret the fact that the storm in D masks the reflections which would trace any change in F. Perhaps the most definite evidence is that of Berkner based on observations by Berkner and Wells (**34**), which show the F reappearing with little abnormality as soon as the fade-out in D ceases to cover overlying phenomena, E also reappearing with some increase in maximum ion density and virtual height. The argument is that the upper layers of low density would recover more slowly, so that unchanged reappearance after recovery of the lower D region implies that no change in fact can have occurred throughout. Berkner quotes the work of Martyn, Munro, Higgs and Williams (**35**) in Australia, which does record diminutions in F during flares, but considers these are random and not interrelated.

If we utilize our present suggestion that some solar flares may result from transient replacement of Planck by Cillié distribution, intensification and weakening in different ultra-violet bands become susceptible of quantitative approach; such differential change may be considerable even without postulating much excess in total energy output. Novel possibilities arise, for instance the possible ascription of the lowest ionosphere to O_2^+ at 12·2 e.V., and of the upper to 15–19 e.V. in the Bates and Massey theory, becomes compatible with the Australian interpretation of the Canberra observations upon F_2, as follows. The reduction of F_2 ionization, known to characterize certain types of disturbance, was found to accompany the D ionization simultaneously with flares and not merely with magnetic storms of delayed corpuscular origin.

Recall that we considered Eddington's Lyman radiation as redistributed into a Planck spectrum of locally enhanced temperature when generated sufficiently deeply, but as emerging in its original capture form during momentary "surfacing" of the convection column's summit. Irregular turbulence of the convection zone is probably incessant, and so the more permanent of the extra-solar ultra-violet excitation may be a response to Case (ii) of Section 4, where $f_2 \to 0$ and τ is large. That the extreme solar flare is not satisfied by such fluctuating windows of Planck type, however hot, is shown by the lack of thermal accompaniment which we quoted; we have suggested that instead it may represent the rarer switch towards Case (i), where $f_2 \to 1$, $\tau \to 0$, in which the Eddington radiation ceases to be redistributed through depth and is generated in local areas so near the photospheric surface as to approach the Cillié distribution. We have shown that in such instances, where f_1 is not too small, thousands-fold intensification can occur in some frequencies. Since the excess Planck radiation at T' was itself a redistribution of the capture spectrum and not the mean photospheric of the whole Sun, the rare

approach to pure capture spectrum when the convection column surges will suppress the excess Planck components into which capture radiation had ordinarily degenerated. Referring to Fig. 1, it is seen that such interchange enhances extremely the 13–14 e.V. end near the head of the continuum, but less so the other end at 16–18 e.V. With comparable energies over the whole band the region around 600 A. actually falls in intensity when the 900 A. is intensified by the change from equivalent Planck to Cillié, the extent depending on the excess T' by which the Planck distribution exhibits its energy, whereas the capture spectrum is "cool" until transformed by reabsorptions. Since any capture continuum carries with it some radiation in the associated series lines, this intensification at the head will bring also Lyman lines 1215–910 A., the fraction which we denoted by f_1 never reaching unity; the 12–13 e.V. energy which has been suggested as possibly responsible for the lowest ionosphere will rise and the 15–19 e.V. energy for the outer F_2 region may even fall simultaneously.

A simplified example will illustrate numerically how in crude model this might give rise to the kind of observation actually recorded by the Australians: any interpretation beyond the main feature would be dependent on secondary effects, such as the lowering of ion densities by expansion. Consider, in accord with the old view that enhanced ultra-violet means a window into hotter subphotospheric depths, a locally distributed area imitating Planck distribution at 9000 deg.; at the middle of our spectral range the ergs per cm.^{2} per sec. is equivalent to (Fig. 1) a 6000 deg. capture spectrum at the chromospheric densities of $n_+n_- = 10^{22}$, of which indeed a Planck distribution at some raised temperature will be the transformed product. Fig. 1 indicates that between 900 A. and 1000 A., where head of continuum and series lines emerge, the pure capture spectrum rises to 10 times the Planck intensity, but at 650 A. falls below it by a factor of 10. If a solar flare implies surges lifting the summit of the convection column towards $\tau = 0$, so that Eddington's excess recombination occurs nearer the surface and the Planck transformation reverts nearly to the pure capture distribution, the 12 e.V. radiation will increase and the 18 e.V. radiation will diminish. The hundred-fold change in our simplified model will be qualified by the unknown extent to which the Planck temperature had been enhanced, and may be small; but it need not vanish, and slight differential effects throughout the several layers may become significant.

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1947 *October* 25.

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AN ATTEMPT TO DETECT THE MAGNETIC FIELD OF A WHITE DWARF

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(Received 1947 October 30)

Summary

A series of slitless spectra of the 11th mag. white dwarf Wolf 1346 has been taken with and without polarizing apparatus to test Blackett's prediction of large magnetic fields in such stars. No systematic Paschen-Back pattern can be detected in the hydrogen lines with an experimental error of the order of 8 A., corresponding to an upper limit of 10^6 gauss. The need for greater spectral purity is emphasized in a discussion of an apparent structure in the lines which is independent of the polarizing analyser.

Blackett* has suggested that the angular momentum of a white dwarf is associated with a magnetic field of several million gauss. Such a field should lead to Paschen-Back splitting of the hydrogen lines of the order of 50 A. which should be perceptible even on quite low dispersion. It was therefore decided to attempt to test the suggestion at the Solar Physics Observatory, Cambridge, using the 36-inch Common reflector (refigured by Burch) or the Newall 25-inch refractor.

The number of known white dwarfs suitable for the experiment is extraordinarily small. The companions of Sirius and Procyon are too near their primaries; 40 Erid. B ($9^m{\cdot}6$) was not accessible at the time of Blackett's prediction. The next star, Wolf 1346 ($11^m{\cdot}3$) was reckoned to be beyond the grasp of the 25-inch refractor with its slit spectrographs, but to be attainable with a 1-prism slitless spectrograph attached to the 36-inch reflector.

This spectrograph has a 2-inch negative collimator of the same focal ratio as the 36-inch mirror. As camera lens a 2-inch f 1 aspherical singlet, figured and kindly lent by Dr C. R. Burch, was used. The mount for this lens with adjustable tilt for the plate-holder was made in the Observatory workshop. The camera reduced the scale on the plate in the ratio 3·6 : 1 compared with direct photography; guiding errors and atmospheric unsteadiness were thus diminished in the same proportion. The combination proved extremely fast and it was possible to register the spectrum of Wolf 1346 on Ilford Zenith plates in exposures of the order of 15 to 20 minutes. The hydrogen lines $H\gamma$, $H\delta$ and $H\epsilon$ could be detected although sky-fog naturally proved to be a rather severe limitation, especially in July when the first successful plates were obtained. Experience showed that when working so close to the sky-fog it was preferable to use rather slower plates with longer exposures but higher contrast. The dispersion attained, 450 A./mm., at $H\gamma$, was considered adequate to reveal the presence or absence of fields of the order of 10^6 gauss.

The spectrograph in its original design permitted guiding on the star by means of its image reflected off the first unsilvered face of the prism. But since it proved impossible to guide on a star fainter than 8th mag. in this way, a small

* *Nature*, **159**, 658, 1947.

area of about 1 cm.^{2} of the prism was silvered; with this, Wolf 1346 could be comfortably seen in the guiding field, but even so it was only used for guiding in R.A.—the direction in which the spectrum could be widened perpendicular to the dispersion. The guiding in declination, carried out on a neighbouring star of 10th mag., was far more critical since (*a*) any error appeared directly in the spectrum contour, (*b*) the tilt of the plate being about 30° a *zero-error* in declination put the whole spectrum out of focus. The field of good definition was in fact extremely small, much more so than for the usual field for direct photography. The collimator lens was suspected as primarily responsible. However, when plenty of light was available and contrasty fine-grain plates were used the definition was satisfactory; it was possible in such circumstances to detect the K line in α Lyrae, which is known to be less than 2 A. wide. (This must not however be taken as a measure of the resolving power which was about 400.)

As polarizing apparatus a sheet of mica quarter-wave plate was used in front of the polaroid placed between the prism and camera lens, with suitable relative orientation. Column 5 of Table I, which lists the material on which this note is based, indicates the orientation used for the various plates; *A* or α will transmit

TABLE I

Diary of Observations of Wolf 1346

Plate No.	Date	Time (U.T.) h m	Exp. (min.)	Polarizer	Emulsion
18	Aug. 10	22 54	82	*A*	Zenith 700
19	Aug. 10	23 55	30	...	Sp. Rapid 270
20	Aug. 11	23 10	120	*B*	Sp. Rapid 270
21	Aug. 12	00 38	30	...	Sp. Rapid 270
22	Aug. 12	22 48	120	β	Sp. Rapid 270
23	Aug. 13	00 57	120	α	Sp. Rapid 270
24	Aug. 13	23 30	240	α	Sp. Rapid 270
25	Sept. 5	21 30	25	...	Empress
26	Sept. 9	22 32·5	17	...	Zenith 700
27	Sept. 9	22 51·5	17	...	Zenith 700
28	Sept. 9	23 15	18	...	Zenith 700

circularly polarized light in one sense, *B* or β in the opposite. *A*, *B* refer to a quarter-wave plate suitable for $H\gamma$; α, β to another suitable for $H\delta$. The relative senses of these two plates were determined in a laboratory test after the measures of the spectra had been completed.

Plates were first examined in a Hilger measuring micrometer and later on microphotometer tracings. In view of the extreme difficulty of measuring the diffuse lines which barely stood out against sky-fog background, the plates were measured a second time (a week later) in a Zeiss micrometer with variable magnification. On this occasion the plates were placed in the micrometer by an independent observer who recorded the plate number.

Since no terrestrial comparison could be impressed on these slitless spectra, a field star with practically the same declination was used as a standard; this 10th mag. star (the same as that used for guiding in declination) showed the hydrogen lines rather more clearly and sharper than Wolf 1346. Normally

three measures were made on the white dwarf lines in each of the two orientations of the plate in the micrometer, and two on the field star.

Table I lists the various measures of Δx, the relative displacement of the hydrogen lines in the spectra of the two stars, in units of 0·01 mm. on the plate; this quantity includes (*a*) the relative displacement in declination of the two stars (calculated to be 0·11 mm. on the scale of the plate), (*b*) curvature of the spectrum lines, (*c*) all displacements due to differences in radial velocity, relativity shift, Zeeman patterns, etc. in the two stars.

TABLE II

Micrometer measures of Wolf 1346 *and Field Star.* Δx *in units of* 0·01 *mm.* (*see text*)

Plate No.	Pol. *A* or α			No Polaroid			Pol. *B* or β		
	Hγ	*Hδ*	*Hε*	*Hγ*	*Hδ*	*Hε*	*Hγ*	*Hδ*	*Hε*
18	19 (1)	17 (1)							
					31 (1)				
19				18 (6)	20 (4)	25 (2)			
				8 (4)	13 (2)	14 (2)			
20							14 (6)	15(2)	21 (2)
								12 (1)	12 (2)
21				23 (6)					
				14 (6)	14 (4)	13 (1)			
22							14 (6)	11 (4)	20 (2)
									8 (2)
23	14 (2)	14 (1)	[29]						
24	[18]	[11]							
	16 (3)	16 (2)			[31]			15 (2)	20 (4)
				20 (12)	20 (4)	25 (2)	14 (12)	11 (5)	10 (4)
				11 (10)	14 (6)	13 (3)			

Figures in round brackets give the weight of the relevant observations.
Figures in square brackets refer to observations considered to be of negligible weight.

Correction for curvature is particularly troublesome as it varies rapidly in the field. It probably accounts for most of the excess of the mean value of Δx over the shift in declination. But all plates quoted in Table II were taken with identical setting of the spectrograph and position of the star in the field; in these conditions a change in Δx which shows correlation with the orientation of the mica plate could only be attributed to the operation of a stellar magnetic field.

Some plates gave signs of structure within the hydrogen lines and although it is felt that such structure might well be attributed to grouping of plate-grain, it is remarkable that it was noted in the white dwarf spectrum and *not* in the field star; (this precludes an explanation in terms of guiding errors*); it was moreover more frequently noted in the short exposures taken without polaroid. In these cases Δx was measured for each apparent component and these are listed separately in Table II, together with the sum of the weights of the measures which were allotted at the time of measurement.

At the foot of Table II are given weighted mean values of Δx, the various "components" being treated as permanent features for this purpose. From the

* The absence in the field star is explicable in terms of plate-grain in that Wolf 1346 was always less densely exposed..

inter-agreement of the two series of measures (with Hilger and Zeiss micrometers) a standard error of $\pm 0{\cdot}018$ mm. for one plate was obtained. It is clear that there is no systematic difference between the spectra with opposite orientations of the polarizing analyser. In other words Paschen-Back splitting of $H\gamma$ is unlikely to exceed 8 A. in Wolf 1346, and can scarcely be a major factor contributing to the observed line-width of about 50 A. in this star.*

Fig. 1 gives microphotometer tracings of a selection of the material used.

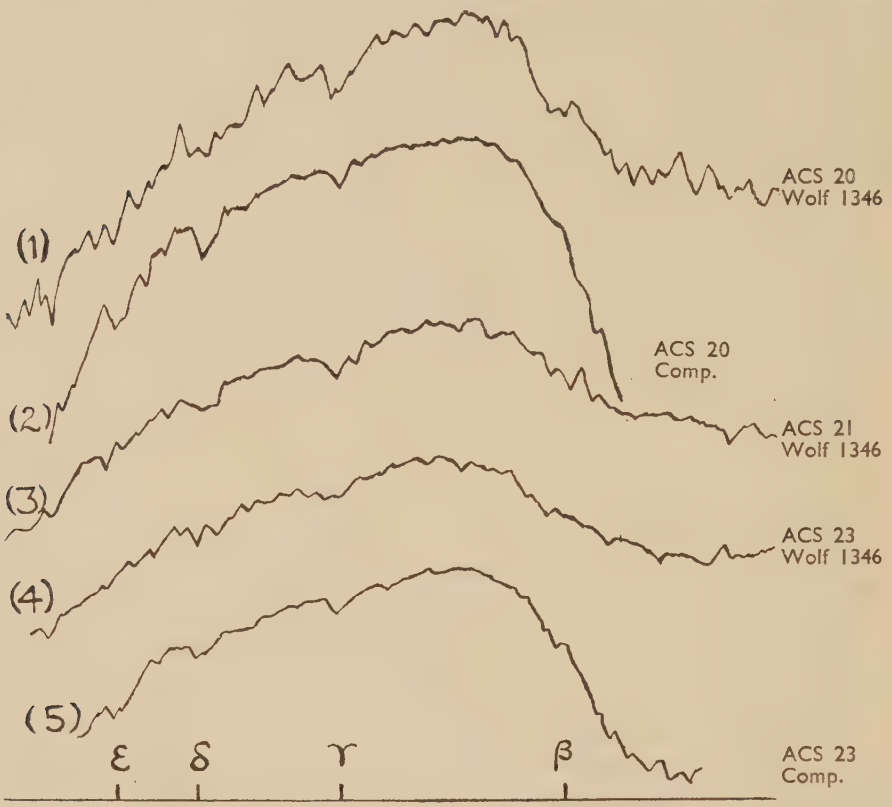

FIG. 1.—*Microphotometer tracings of slitless spectra.*
(1), (4) *Wolf* 1346 *through oppositely polarizing analysers.*
(3) *Wolf* 1346 *without analyser.*
(2), (5) *Field star on same plates as* (1) *and* (4) *respectively.*

The reality of the "components", separated by about 0·1 mm. or 45 to 50 A. (as may be seen in Table II), remains a matter of doubt, which requires further study with higher spectral purity. In order to test the possibility that they represent real features which vary over periods of the order of 1 hour (i.e. less than the exposure times of the plates taken with the analyser) a series of three plates was taken, without polaroid, in immediate succession on the same night, but no systematic trend in the measured Δx or in apparent structure could be detected.

* The test is most suitable to the case where the lines of force are predominantly parallel to the line of sight. If the magnetic axis is at right angles to the line of sight, one might expect a narrowing of the lines with an orientation of the analyser which cuts out both outer σ components. For the pair of plates 18, 20 (orientations "A" and "B") the polaroid was rotated through 90° but no pronounced narrowing of the lines is detectable on either plate. The author is indebted to Professor Blackett for drawing his attention to this point.

40 *Erid. B.*—This star began to be accessible in September and the Burch camera lens was attached to the Newall refractor for an attempt with the Wood aluminized grating spectrograph; this combination gave a dispersion of about 360 A. mm. with rather greater spectral purity than the slitless. A successful 1-hour exposure without polaroid was obtained on 1947 September 25 (showing $H\gamma$ only), but exposures of 1 hour (September 25) and $3\frac{1}{2}$ hours (September 27) through the analyser both failed owing to thick haze forming during exposure.

The writer is deeply indebted to Professor Blackett for his untiring encouragement and assistance in this interesting experiment. It was due to his initiative that the camera lens and the quarter-wave plates which were used were obtained so expeditiously. I am also grateful to Mr L. J. Stanley of the Solar Physics Observatory for providing the short-focus lens with a rigid camera mount, complete with the necessary adjustments, at very short notice.

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MEAN AREAS AND HELIOGRAPHIC LATITUDES OF SUNSPOTS IN THE YEAR 1939

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(Communicated by the Astronomer Royal)

(Received 1947 July 26)

The following results are in continuation of those given in *M.N.*, **102**, 169, 1942, and are derived from the measurement at Greenwich of photographs taken at the Royal Observatories of Greenwich and the Cape and at the Kodaikanal Observatory, India.

Table I gives the mean daily areas of umbrae, whole spots and faculae for each synodic rotation of the Sun included in the year 1939; the means for the year as a whole are included in Table II, which summarizes the yearly values from 1933–39.

Table III gives for each rotation in the year 1939 the mean daily area of the whole spots (corrected for foreshortening) and the mean heliographic latitude of the spotted areas for both northern and southern hemispheres.

The mean heliographic latitude of the entire spotted area and the mean distance from the equator of all spots are also tabulated. The mean values for the year 1939 are included in Table IV.

Tables II and IV are in continuation of similar tables given in *Monthly Notices*: for the years 1874 to 1888—**49**, 381–382; 1889 to 1902—**63**, 465–466; 1901 to 1914—**76**, 402–403; 1913 to 1924—**85**, 1007–1008; 1923 to 1933—**94**, 870–871.

The rotations in Tables I and III are numbered in continuation of Carrington's series.

The chief features of this record for the year 1939 are as follows:—

1. The year was one of many sunspots and their accompanying faculae, but a decline of activity after the high maximum of 1937–38 has very definitely begun. The mean daily sunspot area (corrected for foreshortening) of 1579 for the year 1939 may be compared with the corresponding values of 2019 and 2074 for the years 1938 and 1937 respectively.

The mean daily area derived for periods of a solar synodic rotation showed a considerable variation—from 3093 millionths for rotation 1150 to 821 millionths for rotation 1153. This high value of 3093 millionths (due largely to a giant spot group of mean daily area during its transit of 2504 millionths) has so far been exceeded only once before during the present 11-year cycle, viz. 3209 millionths for rotation 1121 in 1937.

2. On no day of the year were either sunspots or faculae absent.

3. As in each preceding year of the present cycle, with the marked exception of 1937, the southern hemisphere had the greater spotted area. In 1939 this was due, not so much to the larger number of southern spots, but to the five biggest spots of the year each exceeding 1000 millionths in average area, all being in southern latitudes.

4. The number of spot groups lasting for two days or longer was 371, of which 180 were in the northern hemisphere and 191 in the southern. Besides these, there were 134 mostly very small groups seen on one day only; 63 were in the northern hemisphere and 71 in the southern.

TABLE I

Mean Daily Areas

No. of Rotation	Rotation Commenced U.T.	Days Photographed	Projected *			Corrected for Foreshortening †		
			Umbrae	Whole Spots	Faculae	Umbrae	Whole Spots	Faculae
1141	1938 Dec. 28·74	27	311	1575	2301	236	1179	2943
1142	1939 Jan. 25·08	28	287	1403	1944	204	985	2330
1143	Feb. 21·42	27	275	1339	1684	208	1007	2064
1144	Mar. 20·74	27	258	1293	1631	181	907	2028
1145	Apr. 17·03	27	671	3428	2225	497	2595	2826
1146	May 14·27	28	390	1866	2318	295	1420	2924
1147	June 10·47	27	335	1821	2161	245	1333	2666
1148	July 7·67	27	474	2625	1715	342	1864	2102
1149	Aug. 3·88	27	645	3365	1785	457	2374	2253
1150	Aug. 31·12	28	653	4054	1849	509	3093	2259
1151	Sept. 27·38	27	506	2877	1942	367	2103	2331
1152	Oct. 24·67	27	377	2063	1708	246	1340	2026
1153	Nov. 20·98	27	193	1066	1921	147	821	2291
1154	Dec. 18·30	28	289	1432	1392	199	998	1686

* Expressed in millionths of the Sun's disk.

† Expressed in millionths of the Sun's hemisphere.

TABLE II

Year	No. of Days		Mean Daily Areas					
			Projected *			Corrected for Foreshortening †		
	Photographed	Without Spots	Umbrae	Whole Spots	Faculae	Umbrae	Whole Spots	Faculae
1933	365	234	24	120	225	18	88	267
1934	365	145	27	142	297	22	119	354
1935	365	21	152	810	910	117	624	1100
1936	366	0	288	1529	2106	217	1141	2545
1937	365	0	509	2795	2856	375	2074	3505
1938	365	0	501	2706	2578	371	2019	3205
1939	365	0	404	2163	1907	295	1579	2349

* Expressed in millionths of the Sun's disk.

† Expressed in millionths of the Sun's hemisphere.

TABLE III

No. of Rotation	Rotation Commenced U.T.	Spots North of the Equator		Spots South of the Equator		Mean Latitude of Entire Spotted Area	Mean Distance from Equator of all Spots
		Mean Daily Area	Mean Heliographic Latitude	Mean Daily Area	Mean Heliographic Latitude		
			°		°	°	°
1141	1938 Dec. 28·74	409	15·74	770	13·50	− 3·35	14·28
1142	1939 Jan. 25·08	342	12·50	643	13·36	− 4·38	13·06
1143	Feb. 21·42	333	16·19	674	15·56	− 5·07	15·77
1144	Mar. 20·74	455	13·45	452	11·80	+ 0·87	12·63
1145	Apr. 17·03	1688	17·71	907	14·22	+ 6·55	16·49
1146	May 14·27	812	12·93	608	14·97	+ 0·98	13·80
1147	June 10·47	437	12·56	896	10·29	− 2·80	11·03
1148	July 7·67	1196	13·18	668	9·20	+ 5·16	11·75
1149	Aug. 3·88	1342	12·35	1033	14·02	+ 0·88	13·08
1150	Aug. 31·12	226	15·07	2867	13·29	−11·22	13·42
1151	Sept. 27·38	692	17·13	1411	11·34	− 1·98	13·25
1152	Oct. 24·67	527	10·55	813	9·91	− 1·86	10·16
1153	Nov. 20·98	176	23·37	645	10·65	− 3·34	13·38
1154	Dec. 18·30	808	12·50	190	11·33	+ 7·97	12·28

TABLE IV

Year	Days Photographed	Spots North of the Equator		Spots South of the Equator		Mean Latitude of Entire Spotted Area	Mean Distance from Equator of all Spots
		Mean Daily Area	Mean Heliographic Latitude	Mean Daily Area	Mean Heliographic Latitude		
			°		°	°	°
1933	365	86	10·60	2	8·35	+10·23	10·56
1934	365	44	16·45	74	28·09	−11·49	23·75
1934	365 Old Cycle	19	4·10	1	4·37	+ 3·79	4·11
	366 New Cycle	26	25·44	74	28·31	−14·46	27·57
1935	365	205	22·42	419	23·72	− 8·57	23·30
1936	366	463	19·36	678	21·02	− 4·63	20·35
1937	365	1317	17·63	757	15·96	+ 5·36	17·02
1938	365	899	15·55	1120	14·17	− 0·94	14·79
1939	365	649	14·64	931	12·57	− 1·39	13·42

There were 16 spot groups with mean area greater than 500 millionths, and of these, five groups (all south of the equator) well exceeded 1000 millionths. Their dates of central meridian passage and mean areas were:—July 9·1, 1344; September 1·0, 1571; September 10·4, 2504; September 28·0, 1717; October 26·0, 1453.

5. The equatorwards shift of the spot zones continues, the mean weighted latitude of spots 13°·4 being 1°·4 lower than in 1938. Latitudes of individual spots ranged from the equator to 32°. There was a tendency for spots in the northern hemisphere to appear in rather higher latitudes than those in the southern.

Royal Observatory,
Greenwich:
1947 *July* 24.

PRELIMINARY REPORT ON A SEARCH FOR NEW VARIABLE STARS

D. O'Connell, S.J.

(Received 1947 November 27)

Summary

A search for new variable stars is being carried on at Riverview Observatory in a number of regions in the southern sky. A progress report is given here on work in two Milky Way fields. The number of new variables of each type and in different magnitude ranges is compared with the number of known variables and with the number of known variables rediscovered in the course of the search. Tentative conclusions are drawn with regard to the completeness of our survey of variable stars of different types.

Photographic research on variable stars was commenced at Riverview Observatory some years before the war. The programme includes the study of known variable stars and the discovery of new ones. An intensive search for new variable stars has been carried on for many years at a number of observatories, especially at Harvard Observatory, where already over 10,000 variable stars have been discovered. A large proportion of these are faint stars, but there are some thousands brighter than the eleventh magnitude, and it might be thought that by now nearly all the brighter variables had been picked up. It seemed worth while to test this and to try to estimate how near completion is our survey of the various categories of variable stars. The search is still very incomplete. It was started at the end of 1934, but was interrupted by the war and has only now been resumed. Nevertheless a fair number of new variable stars has been discovered.

Seventeen regions, covering most of the Southern Milky Way, from about 7^h to 19^h R.A., are photographed regularly. It will be a long time before the work is completed. However, sufficient has been done to enable some preliminary conclusions to be drawn, and it was thought that a brief account might be of interest. This paper gives a progress report on two of the regions and summarizes the results obtained in these fields up to the beginning of the war.

Most of the plates have been taken with a Zeiss Astrotriplet ($f/4{\cdot}5$, focal length 270 mm.). Eastman 40 plates were used. They were developed in Rodinal, the processing being standardized throughout. With a ten minutes' exposure limiting magnitude on these plates is about $12^m{\cdot}5$. With longer exposures on the Milky Way the fainter stars are usually too crowded for good work. If a variable is much brighter than the 7th magnitude it is difficult to get good magnitude determinations on these plates. The best working range on the plates is from about $7^m{\cdot}0$ to $11^m{\cdot}5$.

The two fields discussed here are centred on α Crucis and η Carinae respectively. The useful area on each plate is about 17° in diameter, say about 250 square degrees. The two areas overlap slightly. The plates were examined for variable stars by the writer in an O'Leary Blink Comparator, which was

completed at the end of 1934. Nine pairs of plates of the α Crucis field were blinked, and five pairs of the η Carinae field. The interval between the plates of a pair varies from 3 hours to 473 days.

The results from the two fields combined are shown in the table. The first column contains the number of known variables in the total area brighter than the eighth photographic magnitude at maximum. They are classified into (1) Cepheids (including cluster type variables), (2) Long Period, (3) Eclipsing, (4) Others (including Novae, irregular variables and those of unknown type, some of which may not be variable). The second column contains the number of each of these types rediscovered in blinking, and the third column the new variables discovered. Columns 4–6 contain similar figures for variables between 8^m and 9^m pg. at maximum; and columns 7–9 and 10–12 for the ranges 9^m–10^m and 10^m–11^m respectively. The known variables are those given in Prager's *Katalog und Ephemeriden der Veränderlichen Sterne* for 1935, the period when the search commenced. In the 1939 *Katalog*, which is the latest available, there are a few additional variables in these fields brighter than the eleventh magnitude at maximum. They include, besides those of the Riverview variables that had then been published, some discovered at Harvard and Leiden.

Variable Stars in α Crucis and η Carinae Fields

		m 8·0 pg.	m 8 – 9	m 9 – 10	m 10 – 11	Total
Cepheids	known	11	9	14	10	44
	rediscovered	10	8	11	7	36
	new	1	2	2	2	7
Long Per.	known	2	3	5	13	23
	rediscovered	2	2	3	4	11
	new	1	0	2	5	8
Eclipsing	known	2	12	14	21	49
	rediscovered	1	9	4	3	17
	new	0	1	4	4	9
Others	known	3	5	8	10	26
	rediscovered	2	3	1	1	7
	new	0	0	0	0	0
Total	known	18	29	41	54	142
	rediscovered	15	22	19	15	71
	new	2	3	8	11	24

Only those new variables that have been completely worked out (i.e. whose range, type, period, etc. have been determined) are given in the table. More than 500 plates were available for the study of each variable, and over 1,000 plates for two variables in the overlapping portion of the two fields. Many more new variables (certain or suspected) in this area remain to be studied. When other plates are searched it is likely that still more variables will be found. It is certain at least that a good many more will be added to those given here.

There were fifty per cent more known variables in the η Carinae field than in the α Crucis field. From five pairs of plates in the former field as many new

variables were discovered as from nine pairs in the other, so that the number of new variables appears to be more or less proportional to the number already known.

It is not worth while making a detailed analysis of these results until the work is further advanced. It is possible nevertheless to draw some tentative conclusions. A comparison of the number of new discoveries with the number of known variables and with the number of rediscoveries, in each category and each magnitude range, suggests that (1) there are a few, but not many, Cepheids brighter than 11^m at maximum yet undiscovered in this area; (2) there are probably a fair number of long-period variables between 9^m and 11^m at maximum yet to be found; (3) the tally of eclipsing variables fainter than the 9th magnitude is even more incomplete.

Riverview College Observatory,
Riverview,
New South Wales:
1947 *November* 21.

THE MASS RATIO AND PARALLAX OF MELB. 4AB FROM MICROMETER MEASURES

W. P. Hirst

(Received 1947 September 3)

Summary

The mass ratio and a new determination of the parallax of the binary Melb. 4AB have been obtained from the results of micrometer measures made at the Union Observatory, Johannesburg, by van den Bos.

The binary star, Melb. 4AB has a 10th magnitude physical companion, C, at a distance of about 32″ in position angle 133°; there is also a 12th magnitude optical companion, D, whose distance in 1940 was about 12″ in position angle 27°.

Between 1928 and 1941, specially careful measures of all the combinations of the four components were made by van den Bos with the 26½ inch refractor at the Union Observatory, Johannesburg with the object of providing material for determining the mass ratio of the close pair whose orbit is now fairly accurately known.

These measures have been used, in conjunction with the orbit elements of AB published by the present author,* to compute the mass ratio of AB and the parallax of the ABC system. The latter turned out, somewhat unexpectedly, to be quite a good determination to judge from the agreement with previously published photographic parallaxes and from the small "mean error".

The method of working adopted was to reduce the measures to rectangular coordinates, taking the x-axis in position angle 133°·5, the mean of the position angles of C from AB. The mean position angle of D from AB is roughly 90° from this, so that each rectangular coordinate depended mainly on one polar coordinate only for both C and D. The A–B positions were computed from the orbit elements.

On comparing the differences between A–C and B–C with A–B in both coordinates, it was found that the former had a tendency to be systematically numerically larger than the latter. Similarly, A–D minus B–D numerically exceeded A–B on the average. Evidently, there had been a tendency, when setting the micrometer wire on A or B, to set too far out and, as A and B are of different magnitudes, the average error of setting is likely to be different in the two cases. This will obviously affect the mass ratio derived from these measures, but there seems to be no way of dealing with it except to give greater weight to those measures that agree with the computed value of A B than to those that do not. Weights were accordingly allocated to the equations of condition on this basis. While this must have reduced the effect of the systematic error to some extent, it is unlikely to have entirely eliminated it, so that the value of the mass ratio found is probably rather more uncertain than the statistical "mean error" indicates.

* *M.N.*, **103**, 344, 1943.

The measures and residuals, together with the details of the computation, will be published in *U.O. Circular* No. 105. The results are as follows:—

Mass Ratio B : AB	$0{\cdot}411 \pm 0{\cdot}027$.
Relative Parallax AB : D	$0{\cdot}130 \pm 0{\cdot}011$.

The paper also gives a new determination of the magnitudes and colour indices of the four components of Melb. 4 by Stoy at the Royal Observatory at the Cape and a summary of data concerning the triple system. The masses and magnitudes of A and B agree closely with the Mass-Luminosity Law.

Cape Town :
1947 *August* 6.

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†What are cosmic rays ? Revised and enlarged American edition. Translated from the French by M. M. Shapiro. 8vo. Chicago, 1945

Brinkmann (D.) :

Johann Ludwig Steiner, ein vergessener Mitbegründer der Zürcher Naturforschenden Gesellschaft. 4to. Frauenfeld, 1946

Davidson (M.) :

The stars and the mind. A study of the impact of astronomical development on human thought. 8vo. London, 1947

Dearborn Observatory of the Northwestern University :

Annals, Vol. VI. 4to. Evanston, 1946

Vol. VI. P. Fox, Measures of double stars.

Evans (D. S.) :

†Frontiers of astronomy. Sigma Introductions to Science, I. 8vo. London, 1946

Hellman (C. D.) :

†The comet of 1577 : its place in the history of astronomy. 8vo. New York, 1944

Kendall (M. G.) :

†Contributions to the study of oscillatory time-series. (National Institute of Economic and Social Research, Occasional Papers, IX.) 8vo. Cambridge, 1946

Le Planetarium :

Guide explicatif : éléments de cosmographie. Deuxième édition. (*J. F. Cox.*) 8vo. Bruxellles, s.d.

London, The Physical Society :

†Reports on progress in physics, Vol. X. 8vo. London, 1946

London, The Royal Society :

Empire Scientific Conference, preliminary report. 8vo. London, 1947

Meudon, Observatoire de Paris, Section d'Astrophysique :

Cartes synoptiques de la chromosphère solaire et catalogue des filaments de la couche supérieure par L. d'Azambuja et R. Servajean. Vol. I, Fasc. VII. 4to. Orléans, 1942

Vol. I, Fasc. VII. Année 1937.

Miller (A. H.) :

Gravimetric surveys of 1944 in New Brunswick. (Geological Survey of Canada, Department of Mines and Resources, Bulletin No. 6.) 8vo. Ottawa, 1946

Mizusawa, International Latitude Service :

Results of the International Latitude Service from 1922-7 to 1935-40. Vol. VIII. By H. Kimura. 4to. Mizusawa, 1940

Moscow, Academy of Sciences of the U.S.S.R. :

The variable stars. Edited by B. W. Kukarkin. Bulletin. Vol. 6, No. 1. 8vo. Moscow, 1946

Princeton University :

Bicentennial Conferences. Series I, Conference I. 8vo. Princeton, 1946

Conference I. The future of nuclear science.

Princeton University Observatory :

Contributions No. 21 : 4to. Princeton, 1946

No. 21. F. B. Wood, Photometric researches. The eclipsing variables : AG Virginis, AR Lacertae, TX Ursae Majoris, VV Orionis, R Canis Majoris, SV Camelopardalis, ST Persei, RY Persei, VZ Hydrae.

Robertson (H. P.) :

*The expanding universe. 8vo. New Haven, 1940

Strömgren (B.) :

*Optical sine-tables giving seven-figure values of $x - \sin x$, with arguments x and $\sin x$. (*B. C. Browne.*) 4to. Copenhaven, 1945

Sverdrup (H. V.), Martin W. Johnson and Richard H. Fleming :

†The oceans : their physics, chemistry and general biology. 8vo. New York, 1946

Vassar College Observatory :

Publication, No. 5. 4to. Poughkeepsie, 1946

No. 5. M. W. Makemson, Maya correlation problem.

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Blackett (P. M. S.):

*The magnetic field of massive rotating bodies. 8vo. London, 1947

Brahe (Tycho):

Tycho Brahe's description of his instruments and scientific work, as given in *Astronomiae Instauratae Mechanica* (Wandesburgi, 1598). Translated and edited by H. Raeder, E. Strömgren and B. Strömgren. 4to. Copenhagen, 1946
(*Miss E. Williamson.*)

Dearborn Observatory of the Northwestern University:

Annals, Vol. V (Dearborn Survey of Faint Red Stars), Part 1C and 5; Part 6; Part 7. 4to. Evanston, 1947

Part 1C and 5. O. J. Lee, G. D. Gore and T. J. Bartlett, Titanium oxide stars in Zones $+40{\cdot}5^\circ$ to $+90^\circ$.

Part 6. O. J. Lee, G. D. Gore and T. J. Bartlett, Summary of titanium oxide stars in the survey from declinations $-4{\cdot}5^\circ$ to $+90^\circ$.

Part 7. O. J. Lee, G. D. Gore and T. J. Bartlett, Carbon stars in Zones $+40^\circ$ to $+90^\circ$.

Delporte (E.):

*Compte-rendu de l'Assemblée restreinte de l'Union Astronomique Internationale, tenu à Copenhagen du 7 au 13 mars 1946. 8vo. Bruxelles, 1947

Douglass (A. E.):

Precision of ring dating in tree-ring chronologies. 8vo. Tucson, 1946

Dufay (M. Jean):

*L'Observatoire de l'Université de Lyon, 1932–1941. 8vo. Lyons, 1942

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On the application of the hydrostatic method of levelling and to determination of vertical movements in the Earth's crust. 8vo. Copenhagen, 1947

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*Geological factors in gravity interpretation, illustrated by evidence from India and Burma. 8vo. London, 1946

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†Vorträge über Kosmische Strahlung. 8vo. Berlin, 1943

Järnefelt (G.):

*On the one-body problem in the expanding universe. 8vo. Copenhagen, 1946

Kant (W.):

†Kant's Cosmogony, edited and translated by W. Hastie. 8vo. Glasgow, 1900

Kirkwood Observatory of the Indiana University:

Publications Nos. 9 and 10. 8vo. Bloomington, 1946

No. 9. L. H. Aller, The atmosphere of 10 Lacertae.
No. 10. D. Bohm and L. H. Aller, The electron velocity distribution in gaseous nebulae and stellar envelopes.

Mitchell (S. A.):

Leander McCormack Observatory of the University of Virginia. 4to. Charlottesville, 1947

Moscow, Academy of Sciences of the U.S.S.R., Institut d'Astronomie Théorique:

Ephémérides des petites planètes, pour l'an 1947. 8vo. Moscow, 1947

Qvist (B.):

*L'Intégration d'un modèle d'étoile. 8vo. Copenhagen, 1946

Sacco (F.):

Origine del sistema solare. Evoluzione cosmica. 8vo. Torino, 1944

Schmidt (O. J.):

*On the law of planetary distances. 8vo. Moscow, 1947
*The cosmogonic significance of the position of the ecliptic plane in the galaxy. 8vo. Moscow, 1947

Waldemeier (M.):

*Sonnenflecken und ihre terrestrischen Wirkungen. 4to. Zürich, 1946

Whiston (W.):

The transits of Venus and Mercury over the Sun at their ascending and descending nodes for two centuries and a half.
(*G. H. Wyatt, Esq.*) Sheet, London, 1723

Zeitschrift für Astrophysik:

Band 19. 8vo. Berlin, 1940

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Becker (Friedrich):
Geschichte der Astronomie. 8vo. Bonn, 1947

Bellamy (Ethel F.):
Index Catalogue of Epicentres for 1931–1935. 8vo. Oxford, 1947

Botley (C. M.):
Polar Lights. 8vo. Tunbridge Wells, 1947

Brinkmann (Donald):
Johann Ludwig Steiner, ein vergessener Mitbegründer der Zürcher Naturforschenden Gesellschaft. 4to. Frauenfeld, 1946

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*Unsolved Problem of Climatic change. 8vo. London, 1947

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†Seismology. 8vo. Cambridge, 1947

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*The transfer of radiation in stellar atmospheres. 8vo. U.S.A., 1947

Comrie (L. J.):
*The application of commercial calculating machines to scientific computing. 8vo. London, 1947

Davidson (M.):
Elements of Mathematical Astronomy. 8vo. London, 1947

Delporte (E.):
*Papyrus Michigan III, 150. 4to. Brussels, 1947

Doig (P.):
An Outline of Stellar Astronomy. 8vo. London, 1947

Eisler (R.):
†The Royal Art of Astrology. 8vo. London, 1947

Ellison (M.):
*Recent advances in the Study of Solar prominences. 8vo. Italy, 1947

Euler (Leonard):
†Opera omnia. 4to. Geneva, 1942 and 1947
Series III. Opera Physica Miscellanea Epistolae. Vol. II.
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Gião (Antonio):
*Sur le magnetisme des masses en rotation. 4to. Paris, 1947

Goldberg (Leo) and Aller (L. H.):
†Atoms, Stars and Nebulae. 8vo. London, 1947

Halley (E.):
A Synopsis of the Astronomy of Comets. 8vo. London, 1705
(*Sir Francis McClean.*)

Hills (G. F. S.):
The formation of Continents by Convection. 8vo. London, 1947

Hiltner (W. A.) and Williams (R. C.):
†Photometric Atlas of Stellar Spectra. 8vo. Ann Arbor, 1946

Iljinsky (I.):
* A simple method for the determination of orbits of Planets and Comets. 8vo. Kiev, 1946

International Astronomical Union:
Quarterly Bulletin on Solar Activity. 4to. Zürich, 1946

Janssen (C. L.):
Fifty years activity of the Urania Observatory. 8vo. Copenhagen, 1947
Publications de l'Observatoire Urania, Copenhague. 4to. Copenhagen, 1928–1946
2nd Series. Nos. 7, 8, 9, 10.
3rd Series. No. 1.

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Observations des Surfaces Planetaires. 8vo. Paris, 1942–1946
Fasicule IX. Année 1924.
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(*R. Jarry Desloges*).

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Die Herkunft der Sterne. 8vo. Stuttgart, 1947
(*Professor O. Heckmann.*)

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*Hosmer's method of azimuth determination on the basis of the azimuth difference of Polaris and another star. 4to. Warsaw, 1946

Khan (M. A. R.):
*Atomic Bombs, the Tektite Problem, and contraterrene Meteorites. 8vo. U.S.A., 1947
*The old Moon in the arms of the New. 8vo. U.S.A., 1945
*A decisive test for the presence of even a highly rarified lunar atmosphere. 8vo. U.S.A., 1946
*The Zodiacal light. 8vo. Secunderabad, 1946
*Muslim Contributions to meteoric Astronomy. 8vo. Secunderabad, 1946

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*On the distribution of titanium oxide and carbon stars in our part of the Milky Way. 8vo. U.S.A., 1946

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The Path of Science. 8vo. New York, 1946

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Solar Eclipse—1945. 4to. Moscow, 1945

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Photometric Atlas of the Solar Spectrum from λ 3612 to λ 8771, with an appendix from λ 3332 to λ 3637. folio, Amsterdam, 1940

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†Philosophiae Naturalis Principia Mathematica. Editio Secunda. 4to. Cantabrigiae, 1713
†Commercium Epistolicum—Johannis Collins. 4to. Londini, 1712

Opik (Ernst Julius):
*Gravitational Source of Energy and Convectional Instability. 8vo. Glückstadt, 1947

Princeton University:
Bicentennial Conferences. Series I.
Conference I. The Future of Nuclear Science. 8vo. Princeton, 1946

Royal Society:
Isaac Newton: Tercentenary Celebrations of the Royal Society, 1946 July 15–19. 8vo. London, 1947

Schmidt (O. J.):
*On the Origin of the Sun's Rotation. 8vo. Moscow, 1947

Shapley (Harlow):
*La gran Galaxia de Sculptor. 4to. Cambridge, Mass., 1947

Slouka (Hubert):
Pohledy do Nebe. 4to. Prague, 1947

Stetson (H. T.):
Sunspots in action. 8vo. New York, 1947

Sullivan (J. W. N.):
Isaac Newton, 1642–1727. 8vo. London, 1938
(*Dr G. Merton.*)

Thomas (Oswald):
Atlas der Sternbilder. 4to. Salzburg, 1945

Tolansky (S.):
†High Resolution Spectroscopy. 8vo. London, 1947

Womersley (J. R.):
*Scientific computing in Great Britain. 8vo. London, 1947
(*Dr L. J. Comrie*).

Woolley (R. van der R.):
†A Key to the Stars. 8vo. Glasgow, 1946

Zinner (Ernst):
*Entstehung und Ausbreitung der Coppennicanishen Lehre. 8vo. Bamberg, 1943
*Der Sternenmantel Kaiser Heinrichs, Himmelskunde und Rechenkunst im alten Bamberg. 8vo. Bamberg, 1939
*Wurde die Räderruhr in Deutschland oder in Italian erfunden ? 8vo. Bonn, 1943
*Zur Ehrenrettung des Simon Marius. 8vo. Leipzig, 1942
*Tiroler Sonnenuhren. 4to. Munich, 1939
*Feinmechanische geräte Meisterwerke der renaissancezeit. 8vo. Berlin, 1943
*Vermischte Aufsätze. 8vo. Bamberg, 1946
*Die Horoscope der Weltantstehung. 4to. Berlin, 1943

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NOTICE TO AUTHORS

1. *Communications.*—Papers must be communicated to the Society by a Fellow. They should be accompanied by a summary at the *beginning* of the paper. Authors are requested to submit MSS. in duplicate, when practicable.

2. *Presentation.*—Authors are allowed considerable latitude, but they are requested to follow the general style and arrangement of *Monthly Notices*. References to literature should be given in the standard form, including a date, for printing either as footnotes or in a numbered list at the end of the paper. Each reference should give the name of the author cited, irrespectively of the occurrence of the name in the text (some latitude being permissible, however, in the case of an author referring to his own work).

3. *Notation.*—Authors should conform closely with the recommendations of Commission 3 of the International Astronomical Union (*Trans. I.A.U.*, Vol. VI, p. 345, 1938). Council has decided to adopt the I.A.U. 4-letter abbreviations for constellations where contraction is desirable (Vol. IV, p. 221, 1932).

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7. *Revised Manuscripts.*—When papers are submitted in revised form it is especially requested that they be accompanied by the original MS.

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8. If a paper is to be read at a meeting it should be received by the Assistant Secretary not later than the previous Friday.

9. When submitting papers authors are requested to indicate whether they will be willing and able to read the paper at the next or some subsequent meeting, and approximately how long they would like to be allotted for speaking.

10. The general issue of postcards giving the programme of each meeting has been resumed. Fellows wishing to receive such cards whether for the Ordinary Meetings or for the Geophysical Discussions or both should notify the Assistant Secretary.

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